Dynamics on Polynomials over Finite Fields

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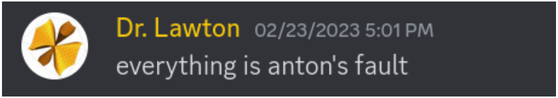

(please don't look at the slide count)

Motivation

Definition

A *planar conic section* is the set of zeroes (variety) in the plane to a polynomial of the form $C_1x^2 + C_2xy + C_3y^2 + C_4x + C_5y + C_6$. For example, the unit circle $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ is a conic section.

We can represent each conic section in the space of conic sections by the coefficients of its corresponding polynomial, or $\mathbb{R}^6$. But since $f(x) = C_1x^2 + \cdots + C_6$ and $\lambda f(x) = \lambda C_1x^2 + \cdots + \lambda C_6$ have the same set of zeroes, we can produce an equivalence relation $\sim$ on $\mathbb{R}^6 - \{0\}$ such that $\vec{x} \sim \vec{y}$ iff $\vec{x} = \lambda\vec{y}$ for some $\lambda \in \mathbb{R}$.

The space $\mathbb{R}^6 - \{0\}/\sim$ is 5 dimensional projective space $\mathbb{R}\mathrm{P}^5$, and is the classical moduli space (space of spaces) of conic sections.

We put a dynamical system on the space of polynomials of arbitrary degree and variable count over finite fields.

Definition

Let $n, d \in \mathbb{N}$ and $\mathbb{F}_q$ be a finite field of order q. We let $\mathrm{GL}_n(\mathbb{F}_q)$ act on $\mathcal{P}_{n,d,q} = \{f \in \mathbb{F}_q[x_1, \ldots, x_n] \mid \mathrm{totaldeg}(f) \leq d\}$ by

$$A \cdot f(\vec{x}) = f(A^{-1}\vec{x}),$$

where $f \in \mathcal{P}_{n,d,q}$ and $A \in \mathrm{GL}_n(\mathbb{F}_q)$.

Theorem

This action is linear, degree preserving, and homogeneity-preserving.

Simple Example

Example

Let $n = d = q = 2$. This is the effect of a generator of $\mathrm{GL}_2(\mathbb{Z}_2)$ on $x^2 + xy$. First compute multiplication on the formal symbols,

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + y \\ y \end{pmatrix},$$

and then compose with the polynomial function:

$$\begin{aligned} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^{-1} \cdot (x^2 + xy) &= (x+y)^2 + (x+y)y \\ &= x^2 + 2xy + y^2 + xy + y^2 \\ &= x^2 + 2xy + xy + 2y^2 \\ &= x^2 + xy. \end{aligned}$$

Note: $2xy = 2y^2 = 0$ since $2 = 0 \bmod 2$.

Invariants

Definition

A polynomial f is *fixed* or an *invariant* if for all $A \in \mathrm{GL}_n(\mathbb{F}_q)$, $A \cdot f = f$.

Theorem

Let $[f]$ *denote the equivalence class of* $f \mod \sim$. *Then,* f *is fixed if and only if* $[f]$ *is fixed. This allows us focus just on the non-projective case for now.*

We found a couple of these invariants by hand. Outside of them, none of the billions of polynomials we checked by code were fixed. We then conjectured that there were no more.

But we were wrong.

Dickson's Theorem [1]

Definition

Denote $|m_1 m_2 \cdots m_n|$ to mean

$$\det\left[x_i^{q^{m_j}}\right]_{ij} = \det\begin{pmatrix} x_1^{q^{m_1}} & x_1^{q^{m_2}} & \dots & x_1^{q^{m_n}} \\ x_2^{q^{m_1}} & x_2^{q^{m_2}} & \dots & x_2^{q^{m_n}} \\ \vdots & \vdots & \ddots & \vdots \\ x_m^{q^{m_1}} & x_m^{q^{m_2}} & \dots & x_m^{q^{m_n}} \end{pmatrix}.$$

Then, define $I_r = |01 \cdots (r-1)\ (r+1)\ \cdots n| / |01 \cdots n-1|$.

Theorem

The set of fixed points $\mathbb{F}_q[x_1, \dots, x_n]^{\mathrm{GL}_n(\mathbb{F}_q)}$ *is a polynomial algebra generated by* $\{I_r : 0 \le r < n\}$.

Example

Let $n = q = 2$. Then, $\mathbb{Z}_2[x, y]^{\mathrm{GL}_n(\mathbb{Z}_2)}$ is generated by

- $I_0 =$

$$|12|/|01| = \frac{\begin{vmatrix} x^2 & x^4 \\ y^2 & y^4 \end{vmatrix}}{\begin{vmatrix} x & x^2 \\ y & y^2 \end{vmatrix}} = \frac{x^2y^4 + x^4y^2}{xy^2 + x^2y} = xy^2 + x^2y \text{ and}$$

- $I_1 =$

$$|02|/|01| = \frac{\begin{vmatrix} x & x^4 \\ y & y^4 \end{vmatrix}}{\begin{vmatrix} x & x^2 \\ y & y^2 \end{vmatrix}} = \frac{xy^4 + x^4y}{xy^2 + x^2y} = x^2 + xy + y^2.$$

Remark

The I_r's in the previous slides match our independently found fixed generators from the previous semester exactly.

Remark

In general, the polynomials generated from these invariants are large and contain many terms.

A small example

The following is the smallest degree polynomial that is fixed when $n = 3$ and $q = 11$.

$$
\begin{aligned}
&x^{1210} + x^{1100}y^{110} + x^{1100}y^{100}z^{10} + x^{1100}y^{90}z^{20} + x^{1100}y^{80}z^{30} + x^{1100}y^{70}z^{40} + x^{1100}y^{60}z^{50} + x^{1100}y^{50}z^{60} + \\
&x^{1100}y^{40}z^{70} + x^{1100}y^{30}z^{80} + x^{1100}y^{20}z^{90} + x^{1100}y^{10}z^{100} + x^{1100}z^{110} - x^{1090}y^{110}z^{10} - x^{1090}y^{100}z^{20} - x^{1090}y^{90}z^{30} - \\
&x^{1090}y^{80}z^{40} - x^{1090}y^{70}z^{50} - x^{1090}y^{60}z^{60} - x^{1090}y^{50}z^{70} - x^{1090}y^{40}z^{80} - x^{1090}y^{30}z^{90} - x^{1090}y^{20}z^{100} - x^{1090}y^{10}z^{110} + \\
&x^{990}y^{220} + 2x^{990}y^{210}z^{10} + 3x^{990}y^{200}z^{20} + 4x^{990}y^{190}z^{30} + 5x^{990}y^{180}z^{40} - 5x^{990}y^{170}z^{50} - 4x^{990}y^{160}z^{60} - 3x^{990}y^{150}z^{70} - \\
&2x^{990}y^{140}z^{80} - x^{990}y^{130}z^{90} + x^{990}y^{110}z^{110} - x^{990}y^{90}z^{130} - 2x^{990}y^{80}z^{140} - 3x^{990}y^{70}z^{150} - 4x^{990}y^{60}z^{160} - 5x^{990}y^{50}z^{170} + \\
&5x^{990}y^{40}z^{180} + 4x^{990}y^{30}z^{190} + 3x^{990}y^{20}z^{200} + 2x^{990}y^{10}z^{210} + x^{990}z^{220} - 2x^{980}y^{220}z^{10} - 4x^{980}y^{210}z^{20} + 5x^{980}y^{200}z^{30} + \\
&3x^{980}y^{190}z^{40} + x^{980}y^{180}z^{50} - x^{980}y^{170}z^{60} - 3x^{980}y^{160}z^{70} - 5x^{980}y^{150}z^{80} + 4x^{980}y^{140}z^{90} + 2x^{980}y^{130}z^{100} + \\
&2x^{980}y^{100}z^{130} + 4x^{980}y^{90}z^{140} - 5x^{980}y^{80}z^{150} - 3x^{980}y^{70}z^{160} - x^{980}y^{60}z^{170} + x^{980}y^{50}z^{180} + 3x^{980}y^{40}z^{190} + 5x^{980}y^{30}z^{200} - \\
&4x^{980}y^{20}z^{210} - 2x^{980}y^{10}z^{220} + x^{970}y^{220}z^{20} + 2x^{970}y^{210}z^{30} + 3x^{970}y^{200}z^{40} + 4x^{970}y^{190}z^{50} + 5x^{970}y^{180}z^{60} - 5x^{970}y^{170}z^{70} - \\
&4x^{970}y^{160}z^{80} - 3x^{970}y^{150}z^{90} - 2x^{970}y^{140}z^{100} - x^{970}y^{130}z^{110} - x^{970}y^{110}z^{130} - 2x^{970}y^{100}z^{140} - 3x^{970}y^{90}z^{150} - \\
&4x^{970}y^{80}z^{160} - 5x^{970}y^{70}z^{170} + 5x^{970}y^{60}z^{180} + 4x^{970}y^{50}z^{190} + 3x^{970}y^{40}z^{200} + 2x^{970}y^{30}z^{210} + x^{970}y^{20}z^{220} + x^{880}y^{330} + \\
&3x^{880}y^{320}z^{10} - 5x^{880}y^{310}z^{20} - x^{880}y^{300}z^{30} + 4x^{880}y^{290}z^{40} - x^{880}y^{280}z^{50} - 5x^{880}y^{270}z^{60} + 3x^{880}y^{260}z^{70} + x^{880}y^{250}z^{80} + \\
&x^{880}y^{220}z^{110} - 3x^{880}y^{200}z^{130} + 3x^{880}y^{190}z^{140} - 4x^{880}y^{180}z^{150} - 2x^{880}y^{170}z^{160} - 2x^{880}y^{160}z^{170} - 4x^{880}y^{150}z^{180} + \\
&3x^{880}y^{140}z^{190} - 3x^{880}y^{130}z^{200} + x^{880}y^{110}z^{220} + x^{880}y^{80}z^{250} + 3x^{880}y^{70}z^{260} - 5x^{880}y^{60}z^{270} - x^{880}y^{50}z^{280} + \\
&4x^{880}y^{40}z^{290} - x^{880}y^{30}z^{300} - 5x^{880}y^{20}z^{310} + 3x^{880}y^{10}z^{320} + x^{880}z^{330} - 3x^{870}y^{330}z^{10} + 2x^{870}y^{320}z^{20} + 4x^{870}y^{310}z^{30} + \\
&3x^{870}y^{300}z^{40} - x^{870}y^{290}z^{50} + 3x^{870}y^{280}z^{60} + 4x^{870}y^{270}z^{70} + 2x^{870}y^{260}z^{80} - 3x^{870}y^{250}z^{90} - 5x^{870}y^{210}z^{130} - \\
&4x^{870}y^{200}z^{140} + 3x^{870}y^{190}z^{150} + 5x^{870}y^{180}z^{160} + 2x^{870}y^{170}z^{170} + 5x^{870}y^{160}z^{180} + 3x^{870}y^{150}z^{190} - 4x^{870}y^{140}z^{200} - \\
&5x^{870}y^{130}z^{210} - 3x^{870}y^{90}z^{250} + 2x^{870}y^{80}z^{260} + 4x^{870}y^{70}z^{270} + 3x^{870}y^{60}z^{280} - x^{870}y^{50}z^{290} + 3x^{870}y^{40}z^{300} + \\
&4x^{870}y^{30}z^{310} + 2x^{870}y^{20}z^{320} - 3x^{870}y^{10}z^{330} + 3x^{860}y^{330}z^{20} - 2x^{860}y^{320}z^{30} - 4x^{860}y^{310}z^{40} - 3x^{860}y^{300}z^{50} + \\
&x^{860}y^{290}z^{60} - 3x^{860}y^{280}z^{70} - 4x^{860}y^{270}z^{80} - 2x^{860}y^{260}z^{90} + 3x^{860}y^{250}z^{100} - 3x^{860}y^{220}z^{130} - x^{860}y^{210}z^{140} - 5
\end{aligned}
$$

$$x^{860}y^{200}z^{150} - 4x^{860}y^{190}z^{160} + 2x^{860}y^{180}z^{170} + 2x^{860}y^{170}z^{180} - 4x^{860}y^{160}z^{190} - 5x^{860}y^{150}z^{200} - x^{860}y^{140}z^{210} -$$
$$3x^{860}y^{130}z^{220} + 3x^{860}y^{100}z^{250} - 2x^{860}y^{90}z^{260} - 4x^{860}y^{80}z^{270} - 3x^{860}y^{70}z^{280} + x^{860}y^{60}z^{290} - 3x^{860}y^{50}z^{300} -$$
$$4x^{860}y^{40}z^{310} - 2x^{860}y^{30}z^{320} + 3x^{860}y^{20}z^{330} - x^{850}y^{330}z^{30} - 3x^{850}y^{320}z^{40} + 5x^{850}y^{310}z^{50} + x^{850}y^{300}z^{60} - 4x^{850}y^{290}z^{70} +$$
$$x^{850}y^{280}z^{80} + 5x^{850}y^{270}z^{90} - 3x^{850}y^{260}z^{100} - x^{850}y^{250}z^{110} + 2x^{850}y^{220}z^{140} - 5x^{850}y^{210}z^{150} + x^{850}y^{200}z^{160} -$$
$$2x^{850}y^{190}z^{170} - 3x^{850}y^{180}z^{180} - 2x^{850}y^{170}z^{190} + x^{850}y^{160}z^{200} - 5x^{850}y^{150}z^{210} + 2x^{850}y^{140}z^{220} - x^{850}y^{110}z^{250} -$$
$$3x^{850}y^{100}z^{260} + 5x^{850}y^{90}z^{270} + x^{850}y^{80}z^{280} - 4x^{850}y^{70}z^{290} + x^{850}y^{60}z^{300} + 5x^{850}y^{50}z^{310} - 3x^{850}y^{40}z^{320} - x^{850}y^{30}z^{330} +$$
$$x^{770}y^{440} + 4x^{770}y^{430}z^{10} - x^{770}y^{420}z^{20} - 2x^{770}y^{410}z^{30} + 2x^{770}y^{400}z^{40} + x^{770}y^{390}z^{50} - 4x^{770}y^{380}z^{60} - x^{770}y^{370}z^{70} +$$
$$x^{770}y^{330}z^{110} + 5x^{770}y^{310}z^{130} + 2x^{770}y^{300}z^{140} - x^{770}y^{290}z^{150} + 4x^{770}y^{280}z^{160} + 3x^{770}y^{270}z^{170} + 4x^{770}y^{260}z^{180} +$$
$$4x^{770}y^{250}z^{190} + x^{770}y^{220}z^{220} + 4x^{770}y^{190}z^{250} + 4x^{770}y^{180}z^{260} + 3x^{770}y^{170}z^{270} + 4x^{770}y^{160}z^{280} - x^{770}y^{150}z^{290} +$$
$$2x^{770}y^{140}z^{300} + 5x^{770}y^{130}z^{310} + x^{770}y^{110}z^{330} - x^{770}y^{70}z^{370} - 4x^{770}y^{60}z^{380} + x^{770}y^{50}z^{390} + 2x^{770}y^{40}z^{400} - 2x^{770}y^{30}z^{410} -$$
$$x^{770}y^{20}z^{420} + 4x^{770}y^{10}z^{430} + x^{770}z^{440} - 4x^{760}y^{440}z^{10} - 5x^{760}y^{430}z^{20} + 4x^{760}y^{420}z^{30} - 3x^{760}y^{410}z^{40} + 3x^{760}y^{400}z^{50} -$$
$$4x^{760}y^{390}z^{60} + 5x^{760}y^{380}z^{70} + 4x^{760}y^{370}z^{80} + x^{760}y^{320}z^{130} + 4x^{760}y^{310}z^{140} - x^{760}y^{300}z^{150} - 2x^{760}y^{290}z^{160} +$$
$$2x^{760}y^{280}z^{170} + x^{760}y^{270}z^{180} - 4x^{760}y^{260}z^{190} - x^{760}y^{250}z^{200} - x^{760}y^{200}z^{250} - 4x^{760}y^{190}z^{260} + x^{760}y^{180}z^{270} +$$
$$2x^{760}y^{170}z^{280} - 2x^{760}y^{160}z^{290} - x^{760}y^{150}z^{300} + 4x^{760}y^{140}z^{310} + x^{760}y^{130}z^{320} + 4x^{760}y^{80}z^{370} + 5x^{760}y^{70}z^{380} -$$
$$4x^{760}y^{60}z^{390} + 3x^{760}y^{50}z^{400} - 3x^{760}y^{40}z^{410} + 4x^{760}y^{30}z^{420} - 5x^{760}y^{20}z^{430} - 4x^{760}y^{10}z^{440} - 5x^{750}y^{440}z^{20} + 2x^{750}y^{430}z^{30} +$$
$$5x^{750}y^{420}z^{40} - x^{750}y^{410}z^{50} + x^{750}y^{400}z^{60} - 5x^{750}y^{390}z^{70} - 2x^{750}y^{380}z^{80} + 5x^{750}y^{370}z^{90} + 5x^{750}y^{330}z^{130} - 3x^{750}y^{320}z^{140} +$$
$$2x^{750}y^{310}z^{150} + 2x^{750}y^{300}z^{160} + x^{750}y^{290}z^{170} + 3x^{750}y^{280}z^{180} + x^{750}y^{270}z^{190} - x^{750}y^{260}z^{200} + x^{750}y^{250}z^{210} +$$
$$x^{750}y^{210}z^{250} - x^{750}y^{200}z^{260} + x^{750}y^{190}z^{270} + 3x^{750}y^{180}z^{280} + x^{750}y^{170}z^{290} + 2x^{750}y^{160}z^{300} + 2x^{750}y^{150}z^{310} -$$
$$3x^{750}y^{140}z^{320} + 5x^{750}y^{130}z^{330} + 5x^{750}y^{90}z^{370} - 2x^{750}y^{80}z^{380} - 5x^{750}y^{70}z^{390} + x^{750}y^{60}z^{400} - x^{750}y^{50}z^{410} + 5x^{750}y^{40}z^{420} +$$
$$2x^{750}y^{30}z^{430} - 5x^{750}y^{20}z^{440} - 4x^{740}y^{440}z^{30} - 5x^{740}y^{430}z^{40} + 4x^{740}y^{420}z^{50} - 3x^{740}y^{410}z^{60} + 3x^{740}y^{400}z^{70} -$$
$$4x^{740}y^{390}z^{80} + 5x^{740}y^{380}z^{90} + 4x^{740}y^{370}z^{100} - 3x^{740}y^{330}z^{140} + 3x^{740}y^{320}z^{150} - 3x^{740}y^{310}z^{160} + 2x^{740}y^{300}z^{170} - 3$$

$x^{740}y^{290}z^{180} + 5x^{740}y^{280}z^{190} + 5x^{740}y^{270}z^{200} - 2x^{740}y^{260}z^{210} - 4x^{740}y^{250}z^{220} - 4x^{740}y^{220}z^{250} - 2x^{740}y^{210}z^{260} +$
$5x^{740}y^{200}z^{270} + 5x^{740}y^{190}z^{280} - 3x^{740}y^{180}z^{290} + 2x^{740}y^{170}z^{300} - 3x^{740}y^{160}z^{310} + 3x^{740}y^{150}z^{320} - 3x^{740}y^{140}z^{330} +$
$4x^{740}y^{100}z^{370} + 5x^{740}y^{90}z^{380} - 4x^{740}y^{80}z^{390} + 3x^{740}y^{70}z^{400} - 3x^{740}y^{60}z^{410} + 4x^{740}y^{50}z^{420} - 5x^{740}y^{40}z^{430} - 4x^{740}y^{30}z^{440} +$
$x^{730}y^{440}z^{40} + 4x^{730}y^{430}z^{50} - x^{730}y^{420}z^{60} - 2x^{730}y^{410}z^{70} + 2x^{730}y^{400}z^{80} + x^{730}y^{390}z^{90} - 4x^{730}y^{380}z^{100} - x^{730}y^{370}z^{110} -$
$3x^{730}y^{330}z^{150} - x^{730}y^{320}z^{160} + 3x^{730}y^{310}z^{170} - 5x^{730}y^{300}z^{180} + 5x^{730}y^{290}z^{190} - 3x^{730}y^{280}z^{200} + x^{730}y^{270}z^{210} +$
$3x^{730}y^{260}z^{220} + 3x^{730}y^{220}z^{260} + x^{730}y^{210}z^{270} - 3x^{730}y^{200}z^{280} + 5x^{730}y^{190}z^{290} - 5x^{730}y^{180}z^{300} + 3x^{730}y^{170}z^{310} -$
$x^{730}y^{160}z^{320} - 3x^{730}y^{150}z^{330} - x^{730}y^{110}z^{370} - 4x^{730}y^{100}z^{380} + x^{730}y^{90}z^{390} + 2x^{730}y^{80}z^{400} - 2x^{730}y^{70}z^{410} - x^{730}y^{60}z^{420} +$
$4x^{730}y^{50}z^{430} + x^{730}y^{40}z^{440} + x^{660}y^{550} + 5x^{660}y^{540}z^{10} + 4x^{660}y^{530}z^{20} + 2x^{660}y^{520}z^{30} + 4x^{660}y^{510}z^{40} + 5x^{660}y^{500}z^{50} +$
$x^{660}y^{490}z^{60} + x^{660}y^{440}z^{110} + x^{660}y^{420}z^{130} + 4x^{660}y^{410}z^{140} + 5x^{660}y^{400}z^{150} - 4x^{660}y^{390}z^{160} - 2x^{660}y^{380}z^{170} -$
$5x^{660}y^{370}z^{180} + x^{660}y^{330}z^{220} - x^{660}y^{300}z^{250} + x^{660}y^{290}z^{260} + 5x^{660}y^{280}z^{270} + 5x^{660}y^{270}z^{280} + x^{660}y^{260}z^{290} -$
$x^{660}y^{250}z^{300} + x^{660}y^{220}z^{330} - 5x^{660}y^{180}z^{370} - 2x^{660}y^{170}z^{380} - 4x^{660}y^{160}z^{390} + 5x^{660}y^{150}z^{400} + 4x^{660}y^{140}z^{410} +$
$x^{660}y^{130}z^{420} + x^{660}y^{110}z^{440} + x^{660}y^{60}z^{490} + 5x^{660}y^{50}z^{500} + 4x^{660}y^{40}z^{510} + 2x^{660}y^{30}z^{520} + 4x^{660}y^{20}z^{530} + 5x^{660}y^{10}z^{540} +$
$x^{660}z^{550} - 5x^{650}y^{550}z^{10} - 3x^{650}y^{540}z^{20} + 2x^{650}y^{530}z^{30} + x^{650}y^{520}z^{40} + 2x^{650}y^{510}z^{50} - 3x^{650}y^{500}z^{60} - 5x^{650}y^{490}z^{70} -$
$2x^{650}y^{430}z^{130} + x^{650}y^{420}z^{140} + 3x^{650}y^{410}z^{150} - 4x^{650}y^{400}z^{160} + 3x^{650}y^{390}z^{170} + x^{650}y^{380}z^{180} - 2x^{650}y^{370}z^{190} +$
$3x^{650}y^{310}z^{250} + 4x^{650}y^{300}z^{260} + x^{650}y^{290}z^{270} - 5x^{650}y^{280}z^{280} + x^{650}y^{270}z^{290} + 4x^{650}y^{260}z^{300} + 3x^{650}y^{250}z^{310} -$
$2x^{650}y^{190}z^{370} + x^{650}y^{180}z^{380} + 3x^{650}y^{170}z^{390} - 4x^{650}y^{160}z^{400} + 3x^{650}y^{150}z^{410} + x^{650}y^{140}z^{420} - 2x^{650}y^{130}z^{430} -$
$5x^{650}y^{70}z^{490} - 3x^{650}y^{60}z^{500} + 2x^{650}y^{50}z^{510} + x^{650}y^{40}z^{520} + 2x^{650}y^{30}z^{530} - 3x^{650}y^{20}z^{540} - 5x^{650}y^{10}z^{550} - x^{640}y^{550}z^{20} -$
$5x^{640}y^{540}z^{30} - 4x^{640}y^{530}z^{40} - 2x^{640}y^{520}z^{50} - 4x^{640}y^{510}z^{60} - 5x^{640}y^{500}z^{70} - x^{640}y^{490}z^{80} + x^{640}y^{440}z^{130} - 3x^{640}y^{430}z^{140} -$
$3x^{640}y^{420}z^{150} + 3x^{640}y^{410}z^{160} - x^{640}y^{400}z^{170} - 5x^{640}y^{390}z^{180} + 5x^{640}y^{380}z^{190} + 3x^{640}y^{370}z^{200} - 3x^{640}y^{320}z^{250} +$
$4x^{640}y^{310}z^{260} - 5x^{640}y^{300}z^{270} + 4x^{640}y^{290}z^{280} + 4x^{640}y^{280}z^{290} - 5x^{640}y^{270}z^{300} + 4x^{640}y^{260}z^{310} - 3x^{640}y^{250}z^{320} +$
$3x^{640}y^{200}z^{370} + 5x^{640}y^{190}z^{380} - 5x^{640}y^{180}z^{390} - x^{640}y^{170}z^{400} + 3x^{640}y^{160}z^{410} - 3x^{640}y^{150}z^{420} - 3x^{640}y^{140}z^{430} +$

$$
\begin{aligned}
&x^{640}y^{130}z^{440} - x^{640}y^{80}z^{490} - 5x^{640}y^{70}z^{500} - 4x^{640}y^{60}z^{510} - 2x^{640}y^{50}z^{520} - 4x^{640}y^{40}z^{530} - 5x^{640}y^{30}z^{540} - x^{640}y^{20}z^{550} + \\
&x^{630}y^{550}z^{30} + 5x^{630}y^{540}z^{40} + 4x^{630}y^{530}z^{50} + 2x^{630}y^{520}z^{60} + 4x^{630}y^{510}z^{70} + 5x^{630}y^{500}z^{80} + x^{630}y^{490}z^{90} - 2x^{630}y^{440}z^{140} - \\
&x^{630}y^{430}z^{150} + 4x^{630}y^{420}z^{160} - x^{630}y^{410}z^{170} - x^{630}y^{400}z^{180} + 4x^{630}y^{390}z^{190} - x^{630}y^{380}z^{200} - 2x^{630}y^{370}z^{210} + x^{630}y^{330}z^{250} - \\
&2x^{630}y^{320}z^{260} + 3x^{630}y^{310}z^{270} + x^{630}y^{300}z^{280} + 5x^{630}y^{290}z^{290} + x^{630}y^{280}z^{300} + 3x^{630}y^{270}z^{310} - 2x^{630}y^{260}z^{320} + \\
&x^{630}y^{250}z^{330} - 2x^{630}y^{210}z^{370} - x^{630}y^{200}z^{380} + 4x^{630}y^{190}z^{390} - x^{630}y^{180}z^{400} - x^{630}y^{170}z^{410} + 4x^{630}y^{160}z^{420} - \\
&x^{630}y^{150}z^{430} - 2x^{630}y^{140}z^{440} + x^{630}y^{90}z^{490} + 5x^{630}y^{80}z^{500} + 4x^{630}y^{70}z^{510} + 2x^{630}y^{60}z^{520} + 4x^{630}y^{50}z^{530} + 5x^{630}y^{40}z^{540} + \\
&x^{630}y^{30}z^{550} + 5x^{620}y^{550}z^{40} + 3x^{620}y^{540}z^{50} - 2x^{620}y^{530}z^{60} - x^{620}y^{520}z^{70} - 2x^{620}y^{510}z^{80} + 3x^{620}y^{500}z^{90} + 5x^{620}y^{490}z^{100} - \\
&4x^{620}y^{440}z^{150} - 3x^{620}y^{430}z^{160} + 3x^{620}y^{420}z^{170} + 5x^{620}y^{410}z^{180} - 4x^{620}y^{400}z^{190} + 4x^{620}y^{390}z^{200} + 4x^{620}y^{380}z^{210} - \\
&5x^{620}y^{370}z^{220} + 4x^{620}y^{330}z^{260} + 2x^{620}y^{320}z^{270} + 3x^{620}y^{310}z^{280} + 2x^{620}y^{300}z^{290} + 2x^{620}y^{290}z^{300} + 3x^{620}y^{280}z^{310} + \\
&2x^{620}y^{270}z^{320} + 4x^{620}y^{260}z^{330} - 5x^{620}y^{220}z^{370} + 4x^{620}y^{210}z^{380} + 4x^{620}y^{200}z^{390} - 4x^{620}y^{190}z^{400} + 5x^{620}y^{180}z^{410} + \\
&3x^{620}y^{170}z^{420} - 3x^{620}y^{160}z^{430} - 4x^{620}y^{150}z^{440} + 5x^{620}y^{100}z^{490} + 3x^{620}y^{90}z^{500} - 2x^{620}y^{80}z^{510} - x^{620}y^{70}z^{520} - \\
&2x^{620}y^{60}z^{530} + 3x^{620}y^{50}z^{540} + 5x^{620}y^{40}z^{550} - x^{610}y^{550}z^{50} - 5x^{610}y^{540}z^{60} - 4x^{610}y^{530}z^{70} - 2x^{610}y^{520}z^{80} - 4x^{610}y^{510}z^{90} - \\
&5x^{610}y^{500}z^{100} - x^{610}y^{490}z^{110} + 4x^{610}y^{440}z^{160} - 2x^{610}y^{430}z^{170} + 5x^{610}y^{420}z^{180} - 3x^{610}y^{410}z^{190} + 5x^{610}y^{400}z^{200} - \\
&2x^{610}y^{390}z^{210} + 4x^{610}y^{380}z^{220} + 5x^{610}y^{330}z^{270} + 3x^{610}y^{320}z^{280} - 2x^{610}y^{310}z^{290} - x^{610}y^{300}z^{300} - 2x^{610}y^{290}z^{310} + \\
&3x^{610}y^{280}z^{320} + 5x^{610}y^{270}z^{330} + 4x^{610}y^{220}z^{380} - 2x^{610}y^{210}z^{390} + 5x^{610}y^{200}z^{400} - 3x^{610}y^{190}z^{410} + 5x^{610}y^{180}z^{420} - \\
&2x^{610}y^{170}z^{430} + 4x^{610}y^{160}z^{440} - x^{610}y^{110}z^{490} - 5x^{610}y^{100}z^{500} - 4x^{610}y^{90}z^{510} - 2x^{610}y^{80}z^{520} - 4x^{610}y^{70}z^{530} - \\
&5x^{610}y^{60}z^{540} - x^{610}y^{50}z^{550} + x^{550}y^{660} - 5x^{550}y^{650}z^{10} - x^{550}y^{640}z^{20} + x^{550}y^{630}z^{30} + 5x^{550}y^{620}z^{40} - x^{550}y^{610}z^{50} + \\
&x^{550}y^{550}z^{110} - 4x^{550}y^{530}z^{130} - 4x^{550}y^{520}z^{140} - x^{550}y^{510}z^{150} + 2x^{550}y^{500}z^{160} - 5x^{550}y^{490}z^{170} + x^{550}y^{440}z^{220} - \\
&2x^{550}y^{410}z^{250} - 5x^{550}y^{400}z^{260} - 5x^{550}y^{390}z^{270} + 4x^{550}y^{380}z^{280} - 4x^{550}y^{370}z^{290} + x^{550}y^{330}z^{330} - 4x^{550}y^{290}z^{370} + \\
&4x^{550}y^{280}z^{380} - 5x^{550}y^{270}z^{390} - 5x^{550}y^{260}z^{400} - 2x^{550}y^{250}z^{410} + x^{550}y^{220}z^{440} - 5x^{550}y^{170}z^{490} + 2x^{550}y^{160}z^{500} - \\
&x^{550}y^{150}z^{510} - 4x^{550}y^{140}z^{520} - 4x^{550}y^{130}z^{530} + x^{550}y^{110}z^{550} - x^{550}y^{50}z^{610} + 5x^{550}y^{40}z^{620} + x^{550}y^{30}z^{630} - x^{550}y^{20}z^{640} - 5
\end{aligned}
$$

$$x^{550}y^{10}z^{650} + x^{550}z^{660} + 5x^{540}y^{660}z^{10} - 3x^{540}y^{650}z^{20} - 5x^{540}y^{640}z^{30} + 5x^{540}y^{630}z^{40} + 3x^{540}y^{620}z^{50} - 5x^{540}y^{610}z^{60} -$$
$$3x^{540}y^{540}z^{130} + 4x^{540}y^{530}z^{140} + 3x^{540}y^{520}z^{150} - 3x^{540}y^{510}z^{160} - 4x^{540}y^{500}z^{170} + 3x^{540}y^{490}z^{180} - 5x^{540}y^{420}z^{250} +$$
$$3x^{540}y^{410}z^{260} + 5x^{540}y^{400}z^{270} - 5x^{540}y^{390}z^{280} - 3x^{540}y^{380}z^{290} + 5x^{540}y^{370}z^{300} + 5x^{540}y^{300}z^{370} - 3x^{540}y^{290}z^{380} -$$
$$5x^{540}y^{280}z^{390} + 5x^{540}y^{270}z^{400} + 3x^{540}y^{260}z^{410} - 5x^{540}y^{250}z^{420} + 3x^{540}y^{180}z^{490} - 4x^{540}y^{170}z^{500} - 3x^{540}y^{160}z^{510} +$$
$$3x^{540}y^{150}z^{520} + 4x^{540}y^{140}z^{530} - 3x^{540}y^{130}z^{540} - 5x^{540}y^{60}z^{610} + 3x^{540}y^{50}z^{620} + 5x^{540}y^{40}z^{630} - 5x^{540}y^{30}z^{640} -$$
$$3x^{540}y^{20}z^{650} + 5x^{540}y^{10}z^{660} + 4x^{530}y^{660}z^{20} + 2x^{530}y^{650}z^{30} - 4x^{530}y^{640}z^{40} + 4x^{530}y^{630}z^{50} - 2x^{530}y^{620}z^{60} -$$
$$4x^{530}y^{610}z^{70} - 4x^{530}y^{550}z^{130} + 4x^{530}y^{540}z^{140} - 4x^{530}y^{530}z^{150} + x^{530}y^{520}z^{160} - 3x^{530}y^{510}z^{170} + x^{530}y^{500}z^{180} +$$
$$5x^{530}y^{490}z^{190} + 5x^{530}y^{430}z^{250} - x^{530}y^{420}z^{260} - 4x^{530}y^{410}z^{270} + 3x^{530}y^{400}z^{280} + 5x^{530}y^{390}z^{290} + 5x^{530}y^{380}z^{300} -$$
$$2x^{530}y^{370}z^{310} - 2x^{530}y^{310}z^{370} + 5x^{530}y^{300}z^{380} + 5x^{530}y^{290}z^{390} + 3x^{530}y^{280}z^{400} - 4x^{530}y^{270}z^{410} - x^{530}y^{260}z^{420} +$$
$$5x^{530}y^{250}z^{430} + 5x^{530}y^{190}z^{490} + x^{530}y^{180}z^{500} - 3x^{530}y^{170}z^{510} + x^{530}y^{160}z^{520} - 4x^{530}y^{150}z^{530} + 4x^{530}y^{140}z^{540} -$$
$$4x^{530}y^{130}z^{550} - 4x^{530}y^{70}z^{610} - 2x^{530}y^{60}z^{620} + 4x^{530}y^{50}z^{630} - 4x^{530}y^{40}z^{640} + 2x^{530}y^{30}z^{650} + 4x^{530}y^{20}z^{660} +$$
$$2x^{520}y^{660}z^{30} + x^{520}y^{650}z^{40} - 2x^{520}y^{640}z^{50} + 2x^{520}y^{630}z^{60} - x^{520}y^{620}z^{70} - 2x^{520}y^{610}z^{80} - 4x^{520}y^{550}z^{140} +$$
$$3x^{520}y^{540}z^{150} + x^{520}y^{530}z^{160} + 2x^{520}y^{520}z^{170} - 4x^{520}y^{510}z^{180} - 4x^{520}y^{500}z^{190} - 5x^{520}y^{490}z^{200} + 2x^{520}y^{440}z^{250} +$$
$$2x^{520}y^{430}z^{260} - x^{520}y^{420}z^{270} + 4x^{520}y^{410}z^{280} + 5x^{520}y^{400}z^{290} - 2x^{520}y^{390}z^{300} - 4x^{520}y^{380}z^{310} + 5x^{520}y^{370}z^{320} +$$
$$5x^{520}y^{320}z^{370} - 4x^{520}y^{310}z^{380} - 2x^{520}y^{300}z^{390} + 5x^{520}y^{290}z^{400} + 4x^{520}y^{280}z^{410} - x^{520}y^{270}z^{420} + 2x^{520}y^{260}z^{430} +$$
$$2x^{520}y^{250}z^{440} - 5x^{520}y^{200}z^{490} - 4x^{520}y^{190}z^{500} - 4x^{520}y^{180}z^{510} + 2x^{520}y^{170}z^{520} + x^{520}y^{160}z^{530} + 3x^{520}y^{150}z^{540} -$$
$$4x^{520}y^{140}z^{550} - 2x^{520}y^{80}z^{610} - x^{520}y^{70}z^{620} + 2x^{520}y^{60}z^{630} - 2x^{520}y^{50}z^{640} + x^{520}y^{40}z^{650} + 2x^{520}y^{30}z^{660} +$$
$$4x^{510}y^{660}z^{40} + 2x^{510}y^{650}z^{50} - 4x^{510}y^{640}z^{60} + 4x^{510}y^{630}z^{70} - 2x^{510}y^{620}z^{80} - 4x^{510}y^{610}z^{90} - x^{510}y^{550}z^{150} -$$
$$3x^{510}y^{540}z^{160} - 3x^{510}y^{530}z^{170} - 4x^{510}y^{520}z^{180} - 2x^{510}y^{510}z^{190} + 5x^{510}y^{500}z^{200} - 3x^{510}y^{490}z^{210} + x^{510}y^{440}z^{260} -$$
$$3x^{510}y^{430}z^{270} + 4x^{510}y^{420}z^{280} + x^{510}y^{410}z^{290} + 3x^{510}y^{400}z^{300} + 2x^{510}y^{390}z^{310} - 4x^{510}y^{380}z^{320} - 4x^{510}y^{370}z^{330} -$$
$$4x^{510}y^{330}z^{370} - 4x^{510}y^{320}z^{380} + 2x^{510}y^{310}z^{390} + 3x^{510}y^{300}z^{400} + x^{510}y^{290}z^{410} + 4x^{510}y^{280}z^{420} - 3x^{510}y^{270}z^{430} +$$
$$x^{510}y^{260}z^{440} - 3x^{510}y^{210}z^{490} + 5x^{510}y^{200}z^{500} - 2x^{510}y^{190}z^{510} - 4x^{510}y^{180}z^{520} - 3x^{510}y^{170}z^{530} - 3x^{510}y^{160}z^{540} -$$

$$x^{510}y^{150}z^{550} - 4x^{510}y^{90}z^{610} - 2x^{510}y^{80}z^{620} + 4x^{510}y^{70}z^{630} - 4x^{510}y^{60}z^{640} + 2x^{510}y^{50}z^{650} + 4x^{510}y^{40}z^{660} + 5x^{500}y^{660}z^{50} -$$
$$3x^{500}y^{650}z^{60} - 5x^{500}y^{640}z^{70} + 5x^{500}y^{630}z^{80} + 3x^{500}y^{620}z^{90} - 5x^{500}y^{610}z^{100} + 2x^{500}y^{550}z^{160} - 4x^{500}y^{540}z^{170} +$$
$$x^{500}y^{530}z^{180} - 4x^{500}y^{520}z^{190} + 5x^{500}y^{510}z^{200} - 5x^{500}y^{500}z^{210} + 5x^{500}y^{490}z^{220} - 3x^{500}y^{440}z^{270} + 2x^{500}y^{430}z^{280} +$$
$$2x^{500}y^{420}z^{290} - x^{500}y^{410}z^{300} + 5x^{500}y^{400}z^{310} + 4x^{500}y^{390}z^{320} + 2x^{500}y^{380}z^{330} + 2x^{500}y^{330}z^{380} + 4x^{500}y^{320}z^{390} +$$
$$5x^{500}y^{310}z^{400} - x^{500}y^{300}z^{410} + 2x^{500}y^{290}z^{420} + 2x^{500}y^{280}z^{430} - 3x^{500}y^{270}z^{440} + 5x^{500}y^{220}z^{490} - 5x^{500}y^{210}z^{500} +$$
$$5x^{500}y^{200}z^{510} - 4x^{500}y^{190}z^{520} + x^{500}y^{180}z^{530} - 4x^{500}y^{170}z^{540} + 2x^{500}y^{160}z^{550} - 5x^{500}y^{100}z^{610} + 3x^{500}y^{90}z^{620} +$$
$$5x^{500}y^{80}z^{630} - 5x^{500}y^{70}z^{640} - 3x^{500}y^{60}z^{650} + 5x^{500}y^{50}z^{660} + x^{490}y^{660}z^{60} - 5x^{490}y^{650}z^{70} - x^{490}y^{640}z^{80} + x^{490}y^{630}z^{90} +$$
$$5x^{490}y^{620}z^{100} - x^{490}y^{610}z^{110} - 5x^{490}y^{550}z^{170} + 3x^{490}y^{540}z^{180} + 5x^{490}y^{530}z^{190} - 5x^{490}y^{520}z^{200} - 3x^{490}y^{510}z^{210} +$$
$$5x^{490}y^{500}z^{220} - x^{490}y^{440}z^{280} + 5x^{490}y^{430}z^{290} + x^{490}y^{420}z^{300} - x^{490}y^{410}z^{310} - 5x^{490}y^{400}z^{320} + x^{490}y^{390}z^{330} + x^{490}y^{330}z^{390} -$$
$$5x^{490}y^{320}z^{400} - x^{490}y^{310}z^{410} + x^{490}y^{300}z^{420} + 5x^{490}y^{290}z^{430} - x^{490}y^{280}z^{440} + 5x^{490}y^{220}z^{500} - 3x^{490}y^{210}z^{510} -$$
$$5x^{490}y^{200}z^{520} + 5x^{490}y^{190}z^{530} + 3x^{490}y^{180}z^{540} - 5x^{490}y^{170}z^{550} - x^{490}y^{110}z^{610} + 5x^{490}y^{100}z^{620} + x^{490}y^{90}z^{630} - x^{490}y^{80}z^{640} -$$
$$5x^{490}y^{70}z^{650} + x^{490}y^{60}z^{660} + x^{440}y^{770} - 4x^{440}y^{760}z^{10} - 5x^{440}y^{750}z^{20} - 4x^{440}y^{740}z^{30} + x^{440}y^{730}z^{40} + x^{440}y^{660}z^{110} +$$
$$x^{440}y^{640}z^{130} - 2x^{440}y^{630}z^{140} - 4x^{440}y^{620}z^{150} + 4x^{440}y^{610}z^{160} + x^{440}y^{550}z^{220} + 2x^{440}y^{520}z^{250} + x^{440}y^{510}z^{260} -$$
$$3x^{440}y^{500}z^{270} - x^{440}y^{490}z^{280} + x^{440}y^{440}z^{330} - 2x^{440}y^{400}z^{370} - 4x^{440}y^{390}z^{380} - 4x^{440}y^{380}z^{390} - 2x^{440}y^{370}z^{400} +$$
$$x^{440}y^{330}z^{440} - x^{440}y^{280}z^{490} - 3x^{440}y^{270}z^{500} + x^{440}y^{260}z^{510} + 2x^{440}y^{250}z^{520} + x^{440}y^{220}z^{550} + 4x^{440}y^{160}z^{610} -$$
$$4x^{440}y^{150}z^{620} - 2x^{440}y^{140}z^{630} + x^{440}y^{130}z^{640} + x^{440}y^{110}z^{660} + x^{440}y^{40}z^{730} - 4x^{440}y^{30}z^{740} - 5x^{440}y^{20}z^{750} - 4x^{440}y^{10}z^{760} +$$
$$x^{440}z^{770} + 4x^{430}y^{770}z^{10} - 5x^{430}y^{760}z^{20} + 2x^{430}y^{750}z^{30} - 5x^{430}y^{740}z^{40} + 4x^{430}y^{730}z^{50} - 2x^{430}y^{650}z^{130} - 3x^{430}y^{640}z^{140} -$$
$$x^{430}y^{630}z^{150} - 3x^{430}y^{620}z^{160} - 2x^{430}y^{610}z^{170} + 5x^{430}y^{530}z^{250} + 2x^{430}y^{520}z^{260} - 3x^{430}y^{510}z^{270} + 2x^{430}y^{500}z^{280} +$$
$$5x^{430}y^{490}z^{290} - 3x^{430}y^{410}z^{370} + x^{430}y^{400}z^{380} + 4x^{430}y^{390}z^{390} + x^{430}y^{380}z^{400} - 3x^{430}y^{370}z^{410} + 5x^{430}y^{290}z^{490} +$$
$$2x^{430}y^{280}z^{500} - 3x^{430}y^{270}z^{510} + 2x^{430}y^{260}z^{520} + 5x^{430}y^{250}z^{530} - 2x^{430}y^{170}z^{610} - 3x^{430}y^{160}z^{620} - x^{430}y^{150}z^{630} -$$
$$3x^{430}y^{140}z^{640} - 2x^{430}y^{130}z^{650} + 4x^{430}y^{50}z^{730} - 5x^{430}y^{40}z^{740} + 2x^{430}y^{30}z^{750} - 5x^{430}y^{20}z^{760} + 4x^{430}y^{10}z^{770} - x^{420}y^{770}z^{20} + 4$$

$$x^{420}y^{760}z^{30} + 5x^{420}y^{750}z^{40} + 4x^{420}y^{740}z^{50} - x^{420}y^{730}z^{60} + x^{420}y^{660}z^{130} + x^{420}y^{650}z^{140} - 3x^{420}y^{640}z^{150} + 4x^{420}y^{630}z^{160} +$$
$$3x^{420}y^{620}z^{170} + 5x^{420}y^{610}z^{180} - 5x^{420}y^{540}z^{250} - x^{420}y^{530}z^{260} - x^{420}y^{520}z^{270} + 4x^{420}y^{510}z^{280} + 2x^{420}y^{500}z^{290} +$$
$$x^{420}y^{490}z^{300} - x^{420}y^{420}z^{370} + 3x^{420}y^{410}z^{380} - 2x^{420}y^{400}z^{390} - 2x^{420}y^{390}z^{400} + 3x^{420}y^{380}z^{410} - x^{420}y^{370}z^{420} + x^{420}y^{300}z^{490} +$$
$$2x^{420}y^{290}z^{500} + 4x^{420}y^{280}z^{510} - x^{420}y^{270}z^{520} - x^{420}y^{260}z^{530} - 5x^{420}y^{250}z^{540} + 5x^{420}y^{180}z^{610} + 3x^{420}y^{170}z^{620} +$$
$$4x^{420}y^{160}z^{630} - 3x^{420}y^{150}z^{640} + x^{420}y^{140}z^{650} + x^{420}y^{130}z^{660} - x^{420}y^{60}z^{730} + 4x^{420}y^{50}z^{740} + 5x^{420}y^{40}z^{750} + 4x^{420}y^{30}z^{760} -$$
$$x^{420}y^{20}z^{770} - 2x^{410}y^{770}z^{30} - 3x^{410}y^{760}z^{40} - x^{410}y^{750}z^{50} - 3x^{410}y^{740}z^{60} - 2x^{410}y^{730}z^{70} + 4x^{410}y^{660}z^{140} + 3x^{410}y^{650}z^{150} +$$
$$3x^{410}y^{640}z^{160} - x^{410}y^{630}z^{170} + 5x^{410}y^{620}z^{180} - 3x^{410}y^{610}z^{190} - 2x^{410}y^{550}z^{250} + 3x^{410}y^{540}z^{260} - 4x^{410}y^{530}z^{270} +$$
$$4x^{410}y^{520}z^{280} + x^{410}y^{510}z^{290} - x^{410}y^{500}z^{300} - x^{410}y^{490}z^{310} - 3x^{410}y^{430}z^{370} + 3x^{410}y^{420}z^{380} + 4x^{410}y^{410}z^{390} +$$
$$3x^{410}y^{400}z^{400} + 4x^{410}y^{390}z^{410} + 3x^{410}y^{380}z^{420} - 3x^{410}y^{370}z^{430} - x^{410}y^{310}z^{490} - x^{410}y^{300}z^{500} + x^{410}y^{290}z^{510} +$$
$$4x^{410}y^{280}z^{520} - 4x^{410}y^{270}z^{530} + 3x^{410}y^{260}z^{540} - 2x^{410}y^{250}z^{550} - 3x^{410}y^{190}z^{610} + 5x^{410}y^{180}z^{620} - x^{410}y^{170}z^{630} +$$
$$3x^{410}y^{160}z^{640} + 3x^{410}y^{150}z^{650} + 4x^{410}y^{140}z^{660} - 2x^{410}y^{70}z^{730} - 3x^{410}y^{60}z^{740} - x^{410}y^{50}z^{750} - 3x^{410}y^{40}z^{760} - 2x^{410}y^{30}z^{770} +$$
$$2x^{400}y^{770}z^{40} + 3x^{400}y^{760}z^{50} + x^{400}y^{750}z^{60} + 3x^{400}y^{740}z^{70} + 2x^{400}y^{730}z^{80} + 5x^{400}y^{660}z^{150} - 4x^{400}y^{650}z^{160} - x^{400}y^{640}z^{170} -$$
$$x^{400}y^{630}z^{180} - 4x^{400}y^{620}z^{190} + 5x^{400}y^{610}z^{200} - 5x^{400}y^{550}z^{260} + 5x^{400}y^{540}z^{270} + 3x^{400}y^{530}z^{280} + 5x^{400}y^{520}z^{290} +$$
$$3x^{400}y^{510}z^{300} + 5x^{400}y^{500}z^{310} - 5x^{400}y^{490}z^{320} - 2x^{400}y^{440}z^{370} + x^{400}y^{430}z^{380} - 2x^{400}y^{420}z^{390} + 3x^{400}y^{410}z^{400} +$$
$$3x^{400}y^{400}z^{410} - 2x^{400}y^{390}z^{420} + x^{400}y^{380}z^{430} - 2x^{400}y^{370}z^{440} - 5x^{400}y^{320}z^{490} + 5x^{400}y^{310}z^{500} + 3x^{400}y^{300}z^{510} +$$
$$5x^{400}y^{290}z^{520} + 3x^{400}y^{280}z^{530} + 5x^{400}y^{270}z^{540} - 5x^{400}y^{260}z^{550} + 5x^{400}y^{200}z^{610} - 4x^{400}y^{190}z^{620} - x^{400}y^{180}z^{630} -$$
$$x^{400}y^{170}z^{640} - 4x^{400}y^{160}z^{650} + 5x^{400}y^{150}z^{660} + 2x^{400}y^{80}z^{730} + 3x^{400}y^{70}z^{740} + x^{400}y^{60}z^{750} + 3x^{400}y^{50}z^{760} + 2x^{400}y^{40}z^{770} +$$
$$x^{390}y^{770}z^{50} - 4x^{390}y^{760}z^{60} - 5x^{390}y^{750}z^{70} - 4x^{390}y^{740}z^{80} + x^{390}y^{730}z^{90} - 4x^{390}y^{660}z^{160} + 3x^{390}y^{650}z^{170} - 5x^{390}y^{640}z^{180} +$$
$$4x^{390}y^{630}z^{190} + 4x^{390}y^{620}z^{200} - 2x^{390}y^{610}z^{210} - 5x^{390}y^{550}z^{270} - 5x^{390}y^{540}z^{280} + 5x^{390}y^{530}z^{290} - 2x^{390}y^{520}z^{300} +$$
$$2x^{390}y^{510}z^{310} + 4x^{390}y^{500}z^{320} + x^{390}y^{490}z^{330} - 4x^{390}y^{440}z^{380} + 4x^{390}y^{430}z^{390} - 2x^{390}y^{420}z^{400} + 4x^{390}y^{410}z^{410} -$$
$$2x^{390}y^{400}z^{420} + 4x^{390}y^{390}z^{430} - 4x^{390}y^{380}z^{440} + x^{390}y^{330}z^{490} + 4x^{390}y^{320}z^{500} + 2x^{390}y^{310}z^{510} - 2x^{390}y^{300}z^{520} + 5$$

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x^{390}y^{290}z^{530} - 5x^{390}y^{280}z^{540} - 5x^{390}y^{270}z^{550} - 2x^{390}y^{210}z^{610} + 4x^{390}y^{200}z^{620} + 4x^{390}y^{190}z^{630} - 5x^{390}y^{180}z^{640} + 3x^{390}y^{170}z^{650} - 4x^{390}y^{160}z^{660} + x^{390}y^{90}z^{730} - 4x^{390}y^{80}z^{740} - 5x^{390}y^{70}z^{750} - 4x^{390}y^{60}z^{760} + x^{390}y^{50}z^{770} - 4x^{380}y^{770}z^{60} + 5x^{380}y^{760}z^{70} - 2x^{380}y^{750}z^{80} + 5x^{380}y^{740}z^{90} - 4x^{380}y^{730}z^{100} - 2x^{380}y^{660}z^{170} + x^{380}y^{650}z^{180} + 5x^{380}y^{640}z^{190} - x^{380}y^{630}z^{200} + 4x^{380}y^{620}z^{210} + 4x^{380}y^{610}z^{220} + 4x^{380}y^{550}z^{280} - 3x^{380}y^{540}z^{290} + 5x^{380}y^{530}z^{300} - 4x^{380}y^{520}z^{310} - 4x^{380}y^{510}z^{320} + 2x^{380}y^{500}z^{330} - 4x^{380}y^{440}z^{390} + x^{380}y^{430}z^{400} + 3x^{380}y^{420}z^{410} + 3x^{380}y^{410}z^{420} + x^{380}y^{400}z^{430} - 4x^{380}y^{390}z^{440} + 2x^{380}y^{330}z^{500} - 4x^{380}y^{320}z^{510} - 4x^{380}y^{310}z^{520} + 5x^{380}y^{300}z^{530} - 3x^{380}y^{290}z^{540} + 4x^{380}y^{280}z^{550} + 4x^{380}y^{220}z^{610} + 4x^{380}y^{210}z^{620} - x^{380}y^{200}z^{630} + 5x^{380}y^{190}z^{640} + x^{380}y^{180}z^{650} - 2x^{380}y^{170}z^{660} - 4x^{380}y^{100}z^{730} + 5x^{380}y^{90}z^{740} - 2x^{380}y^{80}z^{750} + 5x^{380}y^{70}z^{760} - 4x^{380}y^{60}z^{770} - x^{370}y^{770}z^{70} + 4x^{370}y^{760}z^{80} + 5x^{370}y^{750}z^{90} + 4x^{370}y^{740}z^{100} - x^{370}y^{730}z^{110} - 5x^{370}y^{660}z^{180} - 2x^{370}y^{650}z^{190} + 3x^{370}y^{640}z^{200} - 2x^{370}y^{630}z^{210} - 5x^{370}y^{620}z^{220} - 4x^{370}y^{550}z^{290} + 5x^{370}y^{540}z^{300} - 2x^{370}y^{530}z^{310} + 5x^{370}y^{520}z^{320} - 4x^{370}y^{510}z^{330} - 2x^{370}y^{440}z^{400} - 3x^{370}y^{430}z^{410} - x^{370}y^{420}z^{420} - 3x^{370}y^{410}z^{430} - 2x^{370}y^{400}z^{440} - 4x^{370}y^{330}z^{510} + 5x^{370}y^{320}z^{520} - 2x^{370}y^{310}z^{530} + 5x^{370}y^{300}z^{540} - 4x^{370}y^{290}z^{550} - 5x^{370}y^{220}z^{620} - 2x^{370}y^{210}z^{630} + 3x^{370}y^{200}z^{640} - 2x^{370}y^{190}z^{650} - 5x^{370}y^{180}z^{660} - x^{370}y^{110}z^{730} + 4x^{370}y^{100}z^{740} + 5x^{370}y^{90}z^{750} + 4x^{370}y^{80}z^{760} - x^{370}y^{70}z^{770} + x^{330}y^{880} - 3x^{330}y^{870}z^{10} + 3x^{330}y^{860}z^{20} - x^{330}y^{850}z^{30} + x^{330}y^{770}z^{110} + 5x^{330}y^{750}z^{130} - 3x^{330}y^{740}z^{140} - 3x^{330}y^{730}z^{150} + x^{330}y^{660}z^{220} + x^{330}y^{630}z^{250} + 4x^{330}y^{620}z^{260} + 5x^{330}y^{610}z^{270} + x^{330}y^{550}z^{330} - 4x^{330}y^{510}z^{370} + 2x^{330}y^{500}z^{380} + x^{330}y^{490}z^{390} + x^{330}y^{440}z^{440} + x^{330}y^{390}z^{490} + 2x^{330}y^{380}z^{500} - 4x^{330}y^{370}z^{510} + x^{330}y^{330}z^{550} + 5x^{330}y^{270}z^{610} + 4x^{330}y^{260}z^{620} + x^{330}y^{250}z^{630} + x^{330}y^{220}z^{660} - 3x^{330}y^{150}z^{730} - 3x^{330}y^{140}z^{740} + 5x^{330}y^{130}z^{750} + x^{330}y^{110}z^{770} - x^{330}y^{30}z^{850} + 3x^{330}y^{20}z^{860} - 3x^{330}y^{10}z^{870} + x^{330}z^{880} + 3x^{320}y^{880}z^{10} + 2x^{320}y^{870}z^{20} - 2x^{320}y^{860}z^{30} - 3x^{320}y^{850}z^{40} + x^{320}y^{760}z^{130} - 3x^{320}y^{750}z^{140} + 3x^{320}y^{740}z^{150} - x^{320}y^{730}z^{160} - 3x^{320}y^{640}z^{250} - 2x^{320}y^{630}z^{260} + 2x^{320}y^{620}z^{270} + 3x^{320}y^{610}z^{280} + 5x^{320}y^{520}z^{370} - 4x^{320}y^{510}z^{380} + 4x^{320}y^{500}z^{390} - 5x^{320}y^{490}z^{400} - 5x^{320}y^{400}z^{490} + 4x^{320}y^{390}z^{500} - 4x^{320}y^{380}z^{510} + 5x^{320}y^{370}z^{520} + 3x^{320}y^{280}z^{610} + 2x^{320}y^{270}z^{620} - 2x^{320}y^{260}z^{630} - 3x^{320}y^{250}z^{640} - x^{320}y^{160}z^{730} + 3x^{320}y^{150}z^{740} - 3x^{320}y^{140}z^{750} + x^{320}y^{130}z^{760} - 3x^{320}y^{40}z^{850} - 2x^{320}y^{30}z^{860} + 2x^{320}y^{20}z^{870} + 3x^{320}y^{10}z^{880} - 5x^{310}y^{880}z^{20} + 4
$$

$x^{310}y^{870}z^{30} - 4x^{310}y^{860}z^{40} + 5x^{310}y^{850}z^{50} + 5x^{310}y^{770}z^{130} + 4x^{310}y^{760}z^{140} + 2x^{310}y^{750}z^{150} - 3x^{310}y^{740}z^{160} + 3x^{310}y^{730}z^{170} +$
$3x^{310}y^{650}z^{250} + 4x^{310}y^{640}z^{260} + 3x^{310}y^{630}z^{270} + 3x^{310}y^{620}z^{280} - 2x^{310}y^{610}z^{290} - 2x^{310}y^{530}z^{370} - 4x^{310}y^{520}z^{380} +$
$2x^{310}y^{510}z^{390} + 5x^{310}y^{500}z^{400} - x^{310}y^{490}z^{410} - x^{310}y^{410}z^{490} + 5x^{310}y^{400}z^{500} + 2x^{310}y^{390}z^{510} - 4x^{310}y^{380}z^{520} -$
$2x^{310}y^{370}z^{530} - 2x^{310}y^{290}z^{610} + 3x^{310}y^{280}z^{620} + 3x^{310}y^{270}z^{630} + 4x^{310}y^{260}z^{640} + 3x^{310}y^{250}z^{650} + 3x^{310}y^{170}z^{730} -$
$3x^{310}y^{160}z^{740} + 2x^{310}y^{150}z^{750} + 4x^{310}y^{140}z^{760} + 5x^{310}y^{130}z^{770} + 5x^{310}y^{50}z^{850} - 4x^{310}y^{40}z^{860} + 4x^{310}y^{30}z^{870} - 5x^{310}y^{20}z^{880} -$
$x^{300}y^{880}z^{30} + 3x^{300}y^{870}z^{40} - 3x^{300}y^{860}z^{50} + x^{300}y^{850}z^{60} + 2x^{300}y^{770}z^{140} - x^{300}y^{760}z^{150} + 2x^{300}y^{750}z^{160} + 2x^{300}y^{740}z^{170} -$
$5x^{300}y^{730}z^{180} - x^{300}y^{660}z^{250} + 4x^{300}y^{650}z^{260} - 5x^{300}y^{640}z^{270} + x^{300}y^{630}z^{280} + 2x^{300}y^{620}z^{290} - x^{300}y^{610}z^{300} +$
$5x^{300}y^{540}z^{370} + 5x^{300}y^{530}z^{380} - 2x^{300}y^{520}z^{390} + 3x^{300}y^{510}z^{400} - x^{300}y^{500}z^{410} + x^{300}y^{490}z^{420} + x^{300}y^{420}z^{490} - x^{300}y^{410}z^{500} +$
$3x^{300}y^{400}z^{510} - 2x^{300}y^{390}z^{520} + 5x^{300}y^{380}z^{530} + 5x^{300}y^{370}z^{540} - x^{300}y^{300}z^{610} + 2x^{300}y^{290}z^{620} + x^{300}y^{280}z^{630} -$
$5x^{300}y^{270}z^{640} + 4x^{300}y^{260}z^{650} - x^{300}y^{250}z^{660} - 5x^{300}y^{180}z^{730} + 2x^{300}y^{170}z^{740} + 2x^{300}y^{160}z^{750} - x^{300}y^{150}z^{760} +$
$2x^{300}y^{140}z^{770} + x^{300}y^{60}z^{850} - 3x^{300}y^{50}z^{860} + 3x^{300}y^{40}z^{870} - x^{300}y^{30}z^{880} + 4x^{290}y^{880}z^{40} - x^{290}y^{870}z^{50} + x^{290}y^{860}z^{60} -$
$4x^{290}y^{850}z^{70} - x^{290}y^{770}z^{150} - 2x^{290}y^{760}z^{160} + x^{290}y^{750}z^{170} - 3x^{290}y^{740}z^{180} + 5x^{290}y^{730}z^{190} + x^{290}y^{660}z^{260} + x^{290}y^{650}z^{270} +$
$4x^{290}y^{640}z^{280} + 5x^{290}y^{630}z^{290} + 2x^{290}y^{620}z^{300} - 2x^{290}y^{610}z^{310} - 4x^{290}y^{550}z^{370} - 3x^{290}y^{540}z^{380} + 5x^{290}y^{530}z^{390} +$
$5x^{290}y^{520}z^{400} + x^{290}y^{510}z^{410} + 2x^{290}y^{500}z^{420} + 5x^{290}y^{490}z^{430} + 5x^{290}y^{430}z^{490} + 2x^{290}y^{420}z^{500} + x^{290}y^{410}z^{510} +$
$5x^{290}y^{400}z^{520} + 5x^{290}y^{390}z^{530} - 3x^{290}y^{380}z^{540} - 4x^{290}y^{370}z^{550} - 2x^{290}y^{310}z^{610} + 2x^{290}y^{300}z^{620} + 5x^{290}y^{290}z^{630} +$
$4x^{290}y^{280}z^{640} + x^{290}y^{270}z^{650} + x^{290}y^{260}z^{660} + 5x^{290}y^{190}z^{730} - 3x^{290}y^{180}z^{740} + x^{290}y^{170}z^{750} - 2x^{290}y^{160}z^{760} - x^{290}y^{150}z^{770} -$
$4x^{290}y^{70}z^{850} + x^{290}y^{60}z^{860} - x^{290}y^{50}z^{870} + 4x^{290}y^{40}z^{880} - x^{280}y^{880}z^{50} + 3x^{280}y^{870}z^{60} - 3x^{280}y^{860}z^{70} + x^{280}y^{850}z^{80} +$
$4x^{280}y^{770}z^{160} + 2x^{280}y^{760}z^{170} + 3x^{280}y^{750}z^{180} + 5x^{280}y^{740}z^{190} - 3x^{280}y^{730}z^{200} + 5x^{280}y^{660}z^{270} - 5x^{280}y^{650}z^{280} +$
$4x^{280}y^{640}z^{290} + x^{280}y^{630}z^{300} + 3x^{280}y^{620}z^{310} + 3x^{280}y^{610}z^{320} + 4x^{280}y^{550}z^{380} - 5x^{280}y^{540}z^{390} + 3x^{280}y^{530}z^{400} +$
$4x^{280}y^{520}z^{410} + 4x^{280}y^{510}z^{420} + 2x^{280}y^{500}z^{430} - x^{280}y^{490}z^{440} - x^{280}y^{440}z^{490} + 2x^{280}y^{430}z^{500} + 4x^{280}y^{420}z^{510} +$
$4x^{280}y^{410}z^{520} + 3x^{280}y^{400}z^{530} - 5x^{280}y^{390}z^{540} + 4x^{280}y^{380}z^{550} + 3x^{280}y^{320}z^{610} + 3x^{280}y^{310}z^{620} + x^{280}y^{300}z^{630} + 4$

$$x^{280}y^{290}z^{640} - 5x^{280}y^{280}z^{650} + 5x^{280}y^{270}z^{660} - 3x^{280}y^{200}z^{730} + 5x^{280}y^{190}z^{740} + 3x^{280}y^{180}z^{750} + 2x^{280}y^{170}z^{760} +$$
$$4x^{280}y^{160}z^{770} + x^{280}y^{80}z^{850} - 3x^{280}y^{70}z^{860} + 3x^{280}y^{60}z^{870} - x^{280}y^{50}z^{880} - 5x^{270}y^{880}z^{60} + 4x^{270}y^{870}z^{70} - 4x^{270}y^{860}z^{80} +$$
$$5x^{270}y^{850}z^{90} + 3x^{270}y^{770}z^{170} + x^{270}y^{760}z^{180} + x^{270}y^{750}z^{190} + 5x^{270}y^{740}z^{200} + x^{270}y^{730}z^{210} + 5x^{270}y^{660}z^{280} + x^{270}y^{650}z^{290} -$$
$$5x^{270}y^{640}z^{300} + 3x^{270}y^{630}z^{310} + 2x^{270}y^{620}z^{320} + 5x^{270}y^{610}z^{330} - 5x^{270}y^{550}z^{390} + 5x^{270}y^{540}z^{400} - 4x^{270}y^{530}z^{410} -$$
$$x^{270}y^{520}z^{420} - 3x^{270}y^{510}z^{430} - 3x^{270}y^{500}z^{440} - 3x^{270}y^{440}z^{500} - 3x^{270}y^{430}z^{510} - x^{270}y^{420}z^{520} - 4x^{270}y^{410}z^{530} +$$
$$5x^{270}y^{400}z^{540} - 5x^{270}y^{390}z^{550} + 5x^{270}y^{330}z^{610} + 2x^{270}y^{320}z^{620} + 3x^{270}y^{310}z^{630} - 5x^{270}y^{300}z^{640} + x^{270}y^{290}z^{650} +$$
$$5x^{270}y^{280}z^{660} + x^{270}y^{210}z^{730} + 5x^{270}y^{200}z^{740} + x^{270}y^{190}z^{750} + x^{270}y^{180}z^{760} + 3x^{270}y^{170}z^{770} + 5x^{270}y^{90}z^{850} - 4x^{270}y^{80}z^{860} +$$
$$4x^{270}y^{70}z^{870} - 5x^{270}y^{60}z^{880} + 3x^{260}y^{880}z^{70} + 2x^{260}y^{870}z^{80} - 2x^{260}y^{860}z^{90} - 3x^{260}y^{850}z^{100} + 4x^{260}y^{770}z^{180} - 4x^{260}y^{760}z^{190} -$$
$$x^{260}y^{750}z^{200} - 2x^{260}y^{740}z^{210} + 3x^{260}y^{730}z^{220} + x^{260}y^{660}z^{290} + 4x^{260}y^{650}z^{300} + 4x^{260}y^{640}z^{310} - 2x^{260}y^{630}z^{320} +$$
$$4x^{260}y^{620}z^{330} - 5x^{260}y^{550}z^{400} + 3x^{260}y^{540}z^{410} - x^{260}y^{530}z^{420} + 2x^{260}y^{520}z^{430} + x^{260}y^{510}z^{440} + x^{260}y^{440}z^{510} +$$
$$2x^{260}y^{430}z^{520} - x^{260}y^{420}z^{530} + 3x^{260}y^{410}z^{540} - 5x^{260}y^{400}z^{550} + 4x^{260}y^{330}z^{620} - 2x^{260}y^{320}z^{630} + 4x^{260}y^{310}z^{640} +$$
$$4x^{260}y^{300}z^{650} + x^{260}y^{290}z^{660} + 3x^{260}y^{220}z^{730} - 2x^{260}y^{210}z^{740} - x^{260}y^{200}z^{750} - 4x^{260}y^{190}z^{760} + 4x^{260}y^{180}z^{770} -$$
$$3x^{260}y^{100}z^{850} - 2x^{260}y^{90}z^{860} + 2x^{260}y^{80}z^{870} + 3x^{260}y^{70}z^{880} + x^{250}y^{880}z^{80} - 3x^{250}y^{870}z^{90} + 3x^{250}y^{860}z^{100} - x^{250}y^{850}z^{110} +$$
$$4x^{250}y^{770}z^{190} - x^{250}y^{760}z^{200} + x^{250}y^{750}z^{210} - 4x^{250}y^{740}z^{220} - x^{250}y^{660}z^{300} + 3x^{250}y^{650}z^{310} - 3x^{250}y^{640}z^{320} +$$
$$x^{250}y^{630}z^{330} - 2x^{250}y^{550}z^{410} - 5x^{250}y^{540}z^{420} + 5x^{250}y^{530}z^{430} + 2x^{250}y^{520}z^{440} + 2x^{250}y^{440}z^{520} + 5x^{250}y^{430}z^{530} -$$
$$5x^{250}y^{420}z^{540} - 2x^{250}y^{410}z^{550} + x^{250}y^{330}z^{630} - 3x^{250}y^{320}z^{640} + 3x^{250}y^{310}z^{650} - x^{250}y^{300}z^{660} - 4x^{250}y^{220}z^{740} +$$
$$x^{250}y^{210}z^{750} - x^{250}y^{200}z^{760} + 4x^{250}y^{190}z^{770} - x^{250}y^{110}z^{850} + 3x^{250}y^{100}z^{860} - 3x^{250}y^{90}z^{870} + x^{250}y^{80}z^{880} + x^{220}y^{990} -$$
$$2x^{220}y^{980}z^{10} + x^{220}y^{970}z^{20} + x^{220}y^{880}z^{110} - 3x^{220}y^{860}z^{130} + 2x^{220}y^{850}z^{140} + x^{220}y^{770}z^{220} - 4x^{220}y^{740}z^{250} + 3x^{220}y^{730}z^{260} +$$
$$x^{220}y^{660}z^{330} - 5x^{220}y^{620}z^{370} + 4x^{220}y^{610}z^{380} + x^{220}y^{550}z^{440} + 5x^{220}y^{500}z^{490} + 5x^{220}y^{490}z^{500} + x^{220}y^{440}z^{550} +$$
$$4x^{220}y^{380}z^{610} - 5x^{220}y^{370}z^{620} + x^{220}y^{330}z^{660} + 3x^{220}y^{260}z^{730} - 4x^{220}y^{250}z^{740} + x^{220}y^{220}z^{770} + 2x^{220}y^{140}z^{850} -$$
$$3x^{220}y^{130}z^{860} + x^{220}y^{110}z^{880} + x^{220}y^{20}z^{970} - 2x^{220}y^{10}z^{980} + x^{220}z^{990} + 2x^{210}y^{990}z^{10} - 4x^{210}y^{980}z^{20} + 2x^{210}y^{970}z^{30} - 5$$

$x^{210}y^{870}z^{130} - x^{210}y^{860}z^{140} - 5x^{210}y^{850}z^{150} + x^{210}y^{750}z^{250} - 2x^{210}y^{740}z^{260} + x^{210}y^{730}z^{270} - 2x^{210}y^{630}z^{370} +$
$4x^{210}y^{620}z^{380} - 2x^{210}y^{610}z^{390} - 3x^{210}y^{510}z^{490} - 5x^{210}y^{500}z^{500} - 3x^{210}y^{490}z^{510} - 2x^{210}y^{390}z^{610} + 4x^{210}y^{380}z^{620} -$
$2x^{210}y^{370}z^{630} + x^{210}y^{270}z^{730} - 2x^{210}y^{260}z^{740} + x^{210}y^{250}z^{750} - 5x^{210}y^{150}z^{850} - x^{210}y^{140}z^{860} - 5x^{210}y^{130}z^{870} +$
$2x^{210}y^{30}z^{970} - 4x^{210}y^{20}z^{980} + 2x^{210}y^{10}z^{990} + 3x^{200}y^{990}z^{20} + 5x^{200}y^{980}z^{30} + 3x^{200}y^{970}z^{40} - 3x^{200}y^{880}z^{130} -$
$4x^{200}y^{870}z^{140} - 5x^{200}y^{860}z^{150} + x^{200}y^{850}z^{160} - x^{200}y^{760}z^{250} - x^{200}y^{750}z^{260} + 5x^{200}y^{740}z^{270} - 3x^{200}y^{730}z^{280} +$
$3x^{200}y^{640}z^{370} - x^{200}y^{630}z^{380} + 4x^{200}y^{620}z^{390} + 5x^{200}y^{610}z^{400} - 5x^{200}y^{520}z^{490} + 5x^{200}y^{510}z^{500} + 5x^{200}y^{500}z^{510} -$
$5x^{200}y^{490}z^{520} + 5x^{200}y^{400}z^{610} + 4x^{200}y^{390}z^{620} - x^{200}y^{380}z^{630} + 3x^{200}y^{370}z^{640} - 3x^{200}y^{280}z^{730} + 5x^{200}y^{270}z^{740} -$
$x^{200}y^{260}z^{750} - x^{200}y^{250}z^{760} + x^{200}y^{160}z^{850} - 5x^{200}y^{150}z^{860} - 4x^{200}y^{140}z^{870} - 3x^{200}y^{130}z^{880} + 3x^{200}y^{40}z^{970} +$
$5x^{200}y^{30}z^{980} + 3x^{200}y^{20}z^{990} + 4x^{190}y^{990}z^{30} + 3x^{190}y^{980}z^{40} + 4x^{190}y^{970}z^{50} + 3x^{190}y^{880}z^{140} + 3x^{190}y^{870}z^{150} -$
$4x^{190}y^{860}z^{160} - 2x^{190}y^{850}z^{170} + 4x^{190}y^{770}z^{250} - 4x^{190}y^{760}z^{260} + x^{190}y^{750}z^{270} + 5x^{190}y^{740}z^{280} + 5x^{190}y^{730}z^{290} -$
$2x^{190}y^{650}z^{370} + 5x^{190}y^{640}z^{380} + 4x^{190}y^{630}z^{390} - 4x^{190}y^{620}z^{400} - 3x^{190}y^{610}z^{410} + 5x^{190}y^{530}z^{490} - 4x^{190}y^{520}z^{500} -$
$2x^{190}y^{510}z^{510} - 4x^{190}y^{500}z^{520} + 5x^{190}y^{490}z^{530} - 3x^{190}y^{410}z^{610} - 4x^{190}y^{400}z^{620} + 4x^{190}y^{390}z^{630} + 5x^{190}y^{380}z^{640} -$
$2x^{190}y^{370}z^{650} + 5x^{190}y^{290}z^{730} + 5x^{190}y^{280}z^{740} + x^{190}y^{270}z^{750} - 4x^{190}y^{260}z^{760} + 4x^{190}y^{250}z^{770} - 2x^{190}y^{170}z^{850} -$
$4x^{190}y^{160}z^{860} + 3x^{190}y^{150}z^{870} + 3x^{190}y^{140}z^{880} + 4x^{190}y^{50}z^{970} + 3x^{190}y^{40}z^{980} + 4x^{190}y^{30}z^{990} + 5x^{180}y^{990}z^{40} +$
$x^{180}y^{980}z^{50} + 5x^{180}y^{970}z^{60} - 4x^{180}y^{880}z^{150} + 5x^{180}y^{870}z^{160} + 2x^{180}y^{860}z^{170} - 3x^{180}y^{850}z^{180} + 4x^{180}y^{770}z^{260} +$
$x^{180}y^{760}z^{270} + 3x^{180}y^{750}z^{280} - 3x^{180}y^{740}z^{290} - 5x^{180}y^{730}z^{300} - 5x^{180}y^{660}z^{370} + x^{180}y^{650}z^{380} - 5x^{180}y^{640}z^{390} -$
$x^{180}y^{630}z^{400} + 5x^{180}y^{620}z^{410} + 5x^{180}y^{610}z^{420} + 3x^{180}y^{540}z^{490} + x^{180}y^{530}z^{500} - 4x^{180}y^{520}z^{510} - 4x^{180}y^{510}z^{520} +$
$x^{180}y^{500}z^{530} + 3x^{180}y^{490}z^{540} + 5x^{180}y^{420}z^{610} + 5x^{180}y^{410}z^{620} - x^{180}y^{400}z^{630} - 5x^{180}y^{390}z^{640} + x^{180}y^{380}z^{650} -$
$5x^{180}y^{370}z^{660} - 5x^{180}y^{300}z^{730} - 3x^{180}y^{290}z^{740} + 3x^{180}y^{280}z^{750} + x^{180}y^{270}z^{760} + 4x^{180}y^{260}z^{770} - 3x^{180}y^{180}z^{850} +$
$2x^{180}y^{170}z^{860} + 5x^{180}y^{160}z^{870} - 4x^{180}y^{150}z^{880} + 5x^{180}y^{60}z^{970} + x^{180}y^{50}z^{980} + 5x^{180}y^{40}z^{990} - 5x^{170}y^{990}z^{50} - x^{170}y^{980}z^{60} -$
$5x^{170}y^{970}z^{70} - 2x^{170}y^{880}z^{160} + 2x^{170}y^{870}z^{170} + 2x^{170}y^{860}z^{180} - 2x^{170}y^{850}z^{190} + 3x^{170}y^{770}z^{270} + 2x^{170}y^{760}z^{280} +$
$x^{170}y^{750}z^{290} + 2x^{170}y^{740}z^{300} + 3x^{170}y^{730}z^{310} - 2x^{170}y^{660}z^{380} + 3x^{170}y^{650}z^{390} - x^{170}y^{640}z^{400} - x^{170}y^{630}z^{410} + 3$

$$x^{170}y^{620}z^{420} - 2x^{170}y^{610}z^{430} - 5x^{170}y^{550}z^{490} - 4x^{170}y^{540}z^{500} - 3x^{170}y^{530}z^{510} + 2x^{170}y^{520}z^{520} - 3x^{170}y^{510}z^{530} -$$
$$4x^{170}y^{500}z^{540} - 5x^{170}y^{490}z^{550} - 2x^{170}y^{430}z^{610} + 3x^{170}y^{420}z^{620} - x^{170}y^{410}z^{630} - x^{170}y^{400}z^{640} + 3x^{170}y^{390}z^{650} -$$
$$2x^{170}y^{380}z^{660} + 3x^{170}y^{310}z^{730} + 2x^{170}y^{300}z^{740} + x^{170}y^{290}z^{750} + 2x^{170}y^{280}z^{760} + 3x^{170}y^{270}z^{770} - 2x^{170}y^{190}z^{850} +$$
$$2x^{170}y^{180}z^{860} + 2x^{170}y^{170}z^{870} - 2x^{170}y^{160}z^{880} - 5x^{170}y^{70}z^{970} - x^{170}y^{60}z^{980} - 5x^{170}y^{50}z^{990} - 4x^{160}y^{990}z^{60} - 3x^{160}y^{980}z^{70} -$$
$$4x^{160}y^{970}z^{80} - 2x^{160}y^{880}z^{170} + 5x^{160}y^{870}z^{180} - 4x^{160}y^{860}z^{190} + x^{160}y^{850}z^{200} + 4x^{160}y^{770}z^{280} - 2x^{160}y^{760}z^{290} +$$
$$2x^{160}y^{750}z^{300} - 3x^{160}y^{740}z^{310} - x^{160}y^{730}z^{320} - 4x^{160}y^{660}z^{390} - 4x^{160}y^{650}z^{400} + 3x^{160}y^{640}z^{410} + 4x^{160}y^{630}z^{420} -$$
$$3x^{160}y^{620}z^{430} + 4x^{160}y^{610}z^{440} + 2x^{160}y^{550}z^{500} - 3x^{160}y^{540}z^{510} + x^{160}y^{530}z^{520} + x^{160}y^{520}z^{530} - 3x^{160}y^{510}z^{540} +$$
$$2x^{160}y^{500}z^{550} + 4x^{160}y^{440}z^{610} - 3x^{160}y^{430}z^{620} + 4x^{160}y^{420}z^{630} + 3x^{160}y^{410}z^{640} - 4x^{160}y^{400}z^{650} - 4x^{160}y^{390}z^{660} -$$
$$x^{160}y^{320}z^{730} - 3x^{160}y^{310}z^{740} + 2x^{160}y^{300}z^{750} - 2x^{160}y^{290}z^{760} + 4x^{160}y^{280}z^{770} + x^{160}y^{200}z^{850} - 4x^{160}y^{190}z^{860} +$$
$$5x^{160}y^{180}z^{870} - 2x^{160}y^{170}z^{880} - 4x^{160}y^{80}z^{970} - 3x^{160}y^{70}z^{980} - 4x^{160}y^{60}z^{990} - 3x^{150}y^{990}z^{70} - 5x^{150}y^{980}z^{80} -$$
$$3x^{150}y^{970}z^{90} - 4x^{150}y^{880}z^{180} + 3x^{150}y^{870}z^{190} - 5x^{150}y^{860}z^{200} - 5x^{150}y^{850}z^{210} - x^{150}y^{770}z^{290} - x^{150}y^{760}z^{300} +$$
$$2x^{150}y^{750}z^{310} + 3x^{150}y^{740}z^{320} - 3x^{150}y^{730}z^{330} + 5x^{150}y^{660}z^{400} + 3x^{150}y^{650}z^{410} - 3x^{150}y^{640}z^{420} - x^{150}y^{630}z^{430} -$$
$$4x^{150}y^{620}z^{440} - x^{150}y^{550}z^{510} + 3x^{150}y^{540}z^{520} - 4x^{150}y^{530}z^{530} + 3x^{150}y^{520}z^{540} - x^{150}y^{510}z^{550} - 4x^{150}y^{440}z^{620} -$$
$$x^{150}y^{430}z^{630} - 3x^{150}y^{420}z^{640} + 3x^{150}y^{410}z^{650} + 5x^{150}y^{400}z^{660} - 3x^{150}y^{330}z^{730} + 3x^{150}y^{320}z^{740} + 2x^{150}y^{310}z^{750} -$$
$$x^{150}y^{300}z^{760} - x^{150}y^{290}z^{770} - 5x^{150}y^{210}z^{850} - 5x^{150}y^{200}z^{860} + 3x^{150}y^{190}z^{870} - 4x^{150}y^{180}z^{880} - 3x^{150}y^{90}z^{970} -$$
$$5x^{150}y^{80}z^{980} - 3x^{150}y^{70}z^{990} - 2x^{140}y^{990}z^{80} + 4x^{140}y^{980}z^{90} - 2x^{140}y^{970}z^{100} + 3x^{140}y^{880}z^{190} - 4x^{140}y^{870}z^{200} -$$
$$x^{140}y^{860}z^{210} + 2x^{140}y^{850}z^{220} + 2x^{140}y^{770}z^{300} + 4x^{140}y^{760}z^{310} - 3x^{140}y^{750}z^{320} - 3x^{140}y^{740}z^{330} + 4x^{140}y^{660}z^{410} +$$
$$x^{140}y^{650}z^{420} - 3x^{140}y^{640}z^{430} - 2x^{140}y^{630}z^{440} - 4x^{140}y^{550}z^{520} + 4x^{140}y^{540}z^{530} + 4x^{140}y^{530}z^{540} - 4x^{140}y^{520}z^{550} -$$
$$2x^{140}y^{440}z^{630} - 3x^{140}y^{430}z^{640} + x^{140}y^{420}z^{650} + 4x^{140}y^{410}z^{660} - 3x^{140}y^{330}z^{740} - 3x^{140}y^{320}z^{750} + 4x^{140}y^{310}z^{760} +$$
$$2x^{140}y^{300}z^{770} + 2x^{140}y^{220}z^{850} - x^{140}y^{210}z^{860} - 4x^{140}y^{200}z^{870} + 3x^{140}y^{190}z^{880} - 2x^{140}y^{100}z^{970} + 4x^{140}y^{90}z^{980} -$$
$$2x^{140}y^{80}z^{990} - x^{130}y^{990}z^{90} + 2x^{130}y^{980}z^{100} - x^{130}y^{970}z^{110} - 3x^{130}y^{880}z^{200} - 5x^{130}y^{870}z^{210} - 3x^{130}y^{860}z^{220} +$$
$$5x^{130}y^{770}z^{310} + x^{130}y^{760}z^{320} + 5x^{130}y^{750}z^{330} + x^{130}y^{660}z^{420} - 2x^{130}y^{650}z^{430} + x^{130}y^{640}z^{440} - 4x^{130}y^{550}z^{530} - 3$$

$x^{130}y^{540}z^{540} - 4x^{130}y^{530}z^{550} + x^{130}y^{440}z^{640} - 2x^{130}y^{430}z^{650} + x^{130}y^{420}z^{660} + 5x^{130}y^{330}z^{750} + x^{130}y^{320}z^{760} +$
$5x^{130}y^{310}z^{770} - 3x^{130}y^{220}z^{860} - 5x^{130}y^{210}z^{870} - 3x^{130}y^{200}z^{880} - x^{130}y^{110}z^{970} + 2x^{130}y^{100}z^{980} - x^{130}y^{90}z^{990} +$
$x^{110}y^{1100} - x^{110}y^{1090}z^{10} + x^{110}y^{990}z^{110} - x^{110}y^{970}z^{130} + x^{110}y^{880}z^{220} - x^{110}y^{850}z^{250} + x^{110}y^{770}z^{330} - x^{110}y^{730}z^{370} +$
$x^{110}y^{660}z^{440} - x^{110}y^{610}z^{490} + x^{110}y^{550}z^{550} - x^{110}y^{490}z^{610} + x^{110}y^{440}z^{660} - x^{110}y^{370}z^{730} + x^{110}y^{330}z^{770} - x^{110}y^{250}z^{850} +$
$x^{110}y^{220}z^{880} - x^{110}y^{130}z^{970} + x^{110}y^{110}z^{990} - x^{110}y^{10}z^{1090} + x^{110}z^{1100} + x^{100}y^{1100}z^{10} - x^{100}y^{1090}z^{20} + 2x^{100}y^{980}z^{130} -$
$2x^{100}y^{970}z^{140} + 3x^{100}y^{860}z^{250} - 3x^{100}y^{850}z^{260} + 4x^{100}y^{740}z^{370} - 4x^{100}y^{730}z^{380} + 5x^{100}y^{620}z^{490} - 5x^{100}y^{610}z^{500} -$
$5x^{100}y^{500}z^{610} + 5x^{100}y^{490}z^{620} - 4x^{100}y^{380}z^{730} + 4x^{100}y^{370}z^{740} - 3x^{100}y^{260}z^{850} + 3x^{100}y^{250}z^{860} - 2x^{100}y^{140}z^{970} +$
$2x^{100}y^{130}z^{980} - x^{100}y^{20}z^{1090} + x^{100}y^{10}z^{1100} + x^{90}y^{1100}z^{20} - x^{90}y^{1090}z^{30} - x^{90}y^{990}z^{130} + 4x^{90}y^{980}z^{140} - 3x^{90}y^{970}z^{150} -$
$3x^{90}y^{870}z^{250} - 2x^{90}y^{860}z^{260} + 5x^{90}y^{850}z^{270} + 5x^{90}y^{750}z^{370} + 5x^{90}y^{740}z^{380} + x^{90}y^{730}z^{390} + x^{90}y^{630}z^{490} + 3x^{90}y^{620}z^{500} -$
$4x^{90}y^{610}z^{510} - 4x^{90}y^{510}z^{610} + 3x^{90}y^{500}z^{620} + x^{90}y^{490}z^{630} + x^{90}y^{390}z^{730} + 5x^{90}y^{380}z^{740} + 5x^{90}y^{370}z^{750} + 5x^{90}y^{270}z^{850} -$
$2x^{90}y^{260}z^{860} - 3x^{90}y^{250}z^{870} - 3x^{90}y^{150}z^{970} + 4x^{90}y^{140}z^{980} - x^{90}y^{130}z^{990} - x^{90}y^{30}z^{1090} + x^{90}y^{20}z^{1100} + x^{80}y^{1100}z^{30} -$
$x^{80}y^{1090}z^{40} - 2x^{80}y^{990}z^{140} - 5x^{80}y^{980}z^{150} - 4x^{80}y^{970}z^{160} + x^{80}y^{880}z^{250} + 2x^{80}y^{870}z^{260} - 4x^{80}y^{860}z^{270} + x^{80}y^{850}z^{280} +$
$4x^{80}y^{760}z^{370} - 2x^{80}y^{750}z^{380} - 4x^{80}y^{740}z^{390} + 2x^{80}y^{730}z^{400} - x^{80}y^{640}z^{490} + 5x^{80}y^{630}z^{500} - 2x^{80}y^{620}z^{510} - 2x^{80}y^{610}z^{520} -$
$2x^{80}y^{520}z^{610} - 2x^{80}y^{510}z^{620} + 5x^{80}y^{500}z^{630} - x^{80}y^{490}z^{640} + 2x^{80}y^{400}z^{730} - 4x^{80}y^{390}z^{740} - 2x^{80}y^{380}z^{750} + 4x^{80}y^{370}z^{760} +$
$x^{80}y^{280}z^{850} - 4x^{80}y^{270}z^{860} + 2x^{80}y^{260}z^{870} + x^{80}y^{250}z^{880} - 4x^{80}y^{160}z^{970} - 5x^{80}y^{150}z^{980} - 2x^{80}y^{140}z^{990} - x^{80}y^{40}z^{1090} +$
$x^{80}y^{30}z^{1100} + x^{70}y^{1100}z^{40} - x^{70}y^{1090}z^{50} - 3x^{70}y^{990}z^{150} - 3x^{70}y^{980}z^{160} - 5x^{70}y^{970}z^{170} + 3x^{70}y^{880}z^{260} + 4x^{70}y^{870}z^{270} -$
$3x^{70}y^{860}z^{280} - 4x^{70}y^{850}z^{290} - x^{70}y^{770}z^{370} + 5x^{70}y^{760}z^{380} - 5x^{70}y^{750}z^{390} + 3x^{70}y^{740}z^{400} - 2x^{70}y^{730}z^{410} -$
$5x^{70}y^{650}z^{490} - 5x^{70}y^{640}z^{500} + 4x^{70}y^{630}z^{510} - x^{70}y^{620}z^{520} - 4x^{70}y^{610}z^{530} - 4x^{70}y^{530}z^{610} - x^{70}y^{520}z^{620} + 4x^{70}y^{510}z^{630} -$
$5x^{70}y^{500}z^{640} - 5x^{70}y^{490}z^{650} - 2x^{70}y^{410}z^{730} + 3x^{70}y^{400}z^{740} - 5x^{70}y^{390}z^{750} + 5x^{70}y^{380}z^{760} - x^{70}y^{370}z^{770} - 4x^{70}y^{290}z^{850} -$
$3x^{70}y^{280}z^{860} + 4x^{70}y^{270}z^{870} + 3x^{70}y^{260}z^{880} - 5x^{70}y^{170}z^{970} - 3x^{70}y^{160}z^{980} - 3x^{70}y^{150}z^{990} - x^{70}y^{50}z^{1090} + x^{70}y^{40}z^{1100} +$

$$x^{60}y^{1100}z^{50} - x^{60}y^{1090}z^{60} - 4x^{60}y^{990}z^{160} - x^{60}y^{980}z^{170} + 5x^{60}y^{970}z^{180} - 5x^{60}y^{880}z^{270} + 3x^{60}y^{870}z^{280} + x^{60}y^{860}z^{290} +$$
$$x^{60}y^{850}z^{300} - 4x^{60}y^{770}z^{380} - 4x^{60}y^{760}z^{390} + x^{60}y^{750}z^{400} - 3x^{60}y^{740}z^{410} - x^{60}y^{730}z^{420} + x^{60}y^{660}z^{490} - 3x^{60}y^{650}z^{500} -$$
$$4x^{60}y^{640}z^{510} + 2x^{60}y^{630}z^{520} - 2x^{60}y^{620}z^{530} - 5x^{60}y^{610}z^{540} - 5x^{60}y^{540}z^{610} - 2x^{60}y^{530}z^{620} + 2x^{60}y^{520}z^{630} -$$
$$4x^{60}y^{510}z^{640} - 3x^{60}y^{500}z^{650} + x^{60}y^{490}z^{660} - x^{60}y^{420}z^{730} - 3x^{60}y^{410}z^{740} + x^{60}y^{400}z^{750} - 4x^{60}y^{390}z^{760} - 4x^{60}y^{380}z^{770} +$$
$$x^{60}y^{300}z^{850} + x^{60}y^{290}z^{860} + 3x^{60}y^{280}z^{870} - 5x^{60}y^{270}z^{880} + 5x^{60}y^{180}z^{970} - x^{60}y^{170}z^{980} - 4x^{60}y^{160}z^{990} - x^{60}y^{60}z^{1090} +$$
$$x^{60}y^{50}z^{1100} + x^{50}y^{1100}z^{60} - x^{50}y^{1090}z^{70} - 5x^{50}y^{990}z^{170} + x^{50}y^{980}z^{180} + 4x^{50}y^{970}z^{190} - x^{50}y^{880}z^{280} - x^{50}y^{870}z^{290} -$$
$$3x^{50}y^{860}z^{300} + 5x^{50}y^{850}z^{310} + x^{50}y^{770}z^{390} + 3x^{50}y^{760}z^{400} - x^{50}y^{750}z^{410} + 4x^{50}y^{740}z^{420} + 4x^{50}y^{730}z^{430} + 5x^{50}y^{660}z^{500} +$$
$$2x^{50}y^{650}z^{510} - 2x^{50}y^{640}z^{520} + 4x^{50}y^{630}z^{530} + 3x^{50}y^{620}z^{540} - x^{50}y^{610}z^{550} - x^{50}y^{550}z^{610} + 3x^{50}y^{540}z^{620} +$$
$$4x^{50}y^{530}z^{630} - 2x^{50}y^{520}z^{640} + 2x^{50}y^{510}z^{650} + 5x^{50}y^{500}z^{660} + 4x^{50}y^{430}z^{730} + 4x^{50}y^{420}z^{740} - x^{50}y^{410}z^{750} +$$
$$3x^{50}y^{400}z^{760} + x^{50}y^{390}z^{770} + 5x^{50}y^{310}z^{850} - 3x^{50}y^{300}z^{860} - x^{50}y^{290}z^{870} - x^{50}y^{280}z^{880} + 4x^{50}y^{190}z^{970} + x^{50}y^{180}z^{980} -$$
$$5x^{50}y^{170}z^{990} - x^{50}y^{70}z^{1090} + x^{50}y^{60}z^{1100} + x^{40}y^{1100}z^{70} - x^{40}y^{1090}z^{80} + 5x^{40}y^{990}z^{180} + 3x^{40}y^{980}z^{190} + 3x^{40}y^{970}z^{200} +$$
$$4x^{40}y^{880}z^{290} + 3x^{40}y^{870}z^{300} - 4x^{40}y^{860}z^{310} - 3x^{40}y^{850}z^{320} + 2x^{40}y^{770}z^{400} - 3x^{40}y^{760}z^{410} + 5x^{40}y^{750}z^{420} -$$
$$5x^{40}y^{740}z^{430} + x^{40}y^{730}z^{440} + 4x^{40}y^{660}z^{510} + x^{40}y^{650}z^{520} - 4x^{40}y^{640}z^{530} + 5x^{40}y^{630}z^{540} + 5x^{40}y^{620}z^{550} + 5x^{40}y^{550}z^{620} +$$
$$5x^{40}y^{540}z^{630} - 4x^{40}y^{530}z^{640} + x^{40}y^{520}z^{650} + 4x^{40}y^{510}z^{660} + x^{40}y^{440}z^{730} - 5x^{40}y^{430}z^{740} + 5x^{40}y^{420}z^{750} - 3x^{40}y^{410}z^{760} +$$
$$2x^{40}y^{400}z^{770} - 3x^{40}y^{320}z^{850} - 4x^{40}y^{310}z^{860} + 3x^{40}y^{300}z^{870} + 4x^{40}y^{290}z^{880} + 3x^{40}y^{200}z^{970} + 3x^{40}y^{190}z^{980} +$$
$$5x^{40}y^{180}z^{990} - x^{40}y^{80}z^{1090} + x^{40}y^{70}z^{1100} + x^{30}y^{1100}z^{80} - x^{30}y^{1090}z^{90} + 4x^{30}y^{990}z^{190} + 5x^{30}y^{980}z^{200} + 2x^{30}y^{970}z^{210} -$$
$$x^{30}y^{880}z^{300} + 4x^{30}y^{870}z^{310} - 2x^{30}y^{860}z^{320} - x^{30}y^{850}z^{330} - 2x^{30}y^{770}z^{410} + 4x^{30}y^{760}z^{420} + 2x^{30}y^{750}z^{430} - 4x^{30}y^{740}z^{440} +$$
$$2x^{30}y^{660}z^{520} + 2x^{30}y^{650}z^{530} - 5x^{30}y^{640}z^{540} + x^{30}y^{630}z^{550} + x^{30}y^{550}z^{630} - 5x^{30}y^{540}z^{640} + 2x^{30}y^{530}z^{650} + 2x^{30}y^{520}z^{660} -$$
$$4x^{30}y^{440}z^{740} + 2x^{30}y^{430}z^{750} + 4x^{30}y^{420}z^{760} - 2x^{30}y^{410}z^{770} - x^{30}y^{330}z^{850} - 2x^{30}y^{320}z^{860} + 4x^{30}y^{310}z^{870} - x^{30}y^{300}z^{880} +$$
$$2x^{30}y^{210}z^{970} + 5x^{30}y^{200}z^{980} + 4x^{30}y^{190}z^{990} - x^{30}y^{90}z^{1090} + x^{30}y^{80}z^{1100} + x^{20}y^{1100}z^{90} - x^{20}y^{1090}z^{100} + 3x^{20}y^{990}z^{200} - 4$$

$$x^{20}y^{980}z^{210} + x^{20}y^{970}z^{220} - 5x^{20}y^{880}z^{310} + 2x^{20}y^{870}z^{320} + 3x^{20}y^{860}z^{330} - x^{20}y^{770}z^{420} - 5x^{20}y^{760}z^{430} - 5x^{20}y^{750}z^{440} + 4x^{20}y^{660}z^{530} - 3x^{20}y^{650}z^{540} - x^{20}y^{640}z^{550} - x^{20}y^{550}z^{640} - 3x^{20}y^{540}z^{650} + 4x^{20}y^{530}z^{660} - 5x^{20}y^{440}z^{750} - 5x^{20}y^{430}z^{760} - x^{20}y^{420}z^{770} + 3x^{20}y^{330}z^{860} + 2x^{20}y^{320}z^{870} - 5x^{20}y^{310}z^{880} + x^{20}y^{220}z^{970} - 4x^{20}y^{210}z^{980} + 3x^{20}y^{200}z^{990} - x^{20}y^{100}z^{1090} + x^{20}y^{90}z^{1100} + x^{10}y^{1100}z^{100} - x^{10}y^{1090}z^{110} + 2x^{10}y^{990}z^{210} - 2x^{10}y^{980}z^{220} + 3x^{10}y^{880}z^{320} - 3x^{10}y^{870}z^{330} + 4x^{10}y^{770}z^{430} - 4x^{10}y^{760}z^{440} + 5x^{10}y^{660}z^{540} - 5x^{10}y^{650}z^{550} - 5x^{10}y^{550}z^{650} + 5x^{10}y^{540}z^{660} - 4x^{10}y^{440}z^{760} + 4x^{10}y^{430}z^{770} - 3x^{10}y^{330}z^{870} + 3x^{10}y^{320}z^{880} - 2x^{10}y^{220}z^{980} + 2x^{10}y^{210}z^{990} - x^{10}y^{110}z^{1090} + x^{10}y^{100}z^{1100} + y^{1210} + y^{1100}z^{110} + y^{990}z^{220} + y^{880}z^{330} + y^{770}z^{440} + y^{660}z^{550} + y^{550}z^{660} + y^{440}z^{770} + y^{330}z^{880} + y^{220}z^{990} + y^{110}z^{1100} + z^{1210}$$

Fixed Point Function

Remaining Question: How many fixed points exist in $\mathcal{P}_{n,d,q}$?

Definition

Define $\mathcal{F} : \mathbb{N}_+ \times \mathbb{N} \times \{p^m : p \text{ prime}, m \in \mathbb{N}_+\} \to \mathbb{N}$ by

$$\mathcal{F}(n,d,q) = \left|\mathcal{P}_{n,d,q}^{\mathrm{GL}_n(\mathbb{F}_q)}\right| = |\{f \in \mathcal{P}_{n,d,q} : f \text{ fixed}\}|,$$

the number of fixed points in $\mathcal{P}_{n,d,q}$.

Theorem

Let $D_{n,q}(i)$ be the number of fixed $I^{\vec{\alpha}} = \prod_{k=0}^{n-1} I_k^{\alpha_k}$ I-monomials of x-totaldegree i for each $1 \leq i \leq d$. Then, there is a closed form for $\mathcal{F}$ in the $D_{n,q}(i)$'s:

$$\mathcal{F}(n,d,q) = q^{\sum_{i=0}^{d} D_{n,q}(i)}.$$

Computed Examples of Number of Fixed Points

- $\mathcal{F}(2,2,2) = 2$
- $\mathcal{F}(10,2,2) = 12288$
- $\mathcal{F}(20,2,2) = 16,492,674,416,640$
- $\mathcal{F}(30,2,2) >$ a quintillion (it overflowed)
- $\mathcal{F}(20,2,3) = 4,374$
- $\mathcal{F}(50,3,3) = 13,122$
- $\mathcal{F}(80,3,3) = 62,762,119,218$
- $\mathcal{F}(400,3,5) = 381,469,726,562,500$

Two Variable Case

Theorem

The number of fixed I-monomials of total x-degree d in $\mathbb{F}_q[x_1, \ldots, x_n]$ is exactly

$$D_{2,q}(d) = \begin{cases} \left(\left\lfloor \frac{k+q}{q} \right\rfloor - \left\lfloor \frac{k+q}{q+1} \right\rfloor \right) & \text{if } d = k(q-1), k \in \mathbb{N} \\ 0 & \text{otherwise.} \end{cases}$$

It does not seem likely that there is a closed-form solution when $n > 2$ based on similar combinatorial problems.

More Variables

Definition

We define a mapping of the I-monomials into $\mathbb{N}^2$ by

$$I^{\vec{\alpha}} \mapsto \left(\sum_{i=0}^{n-1} \alpha_i q^i, \text{totaldeg}_{\vec{x}}(I^{\vec{\alpha}}) \right).$$

Counting the number of invariants under a certain total degree d then resolves to finding the number of points in the image that lie "along and under the line $y = d$".

Theorem

Let $d \in \mathbb{N}$ be our target total degree. Then the only I-monomials with total degree d correspond to points on the lines
$\left\{(\lambda, Cq^n - \lambda) \in \mathbb{N}^2 : \lambda \in \mathbb{N}, \left\lfloor \frac{d}{q^{n-1}} \right\rfloor \leq C \leq \left\lfloor \frac{d}{q^n - q^{n-1}} \right\rfloor \right\}$.

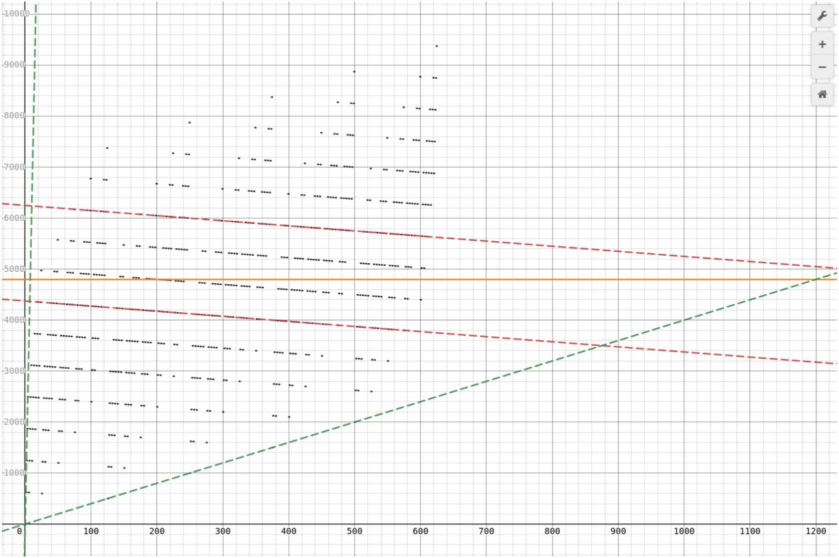

Figure: Bounding C-lines (red) for total degree (orange) and fastest + slowest growing degrees (green) ($q = 5$, $n = 4$)

Full Orbits

Definition

The *orbit* of a polynomial f is $\mathrm{Orb}(f) = \{A \cdot f : A \in \mathrm{GL}_n(\mathbb{F}_q)\}$. An orbit is *full* if it has the same cardinality as $\mathrm{GL}_n(\mathbb{F}_q)$.

Theorem

There is a full orbit in $\mathcal{P}_{n,d,q}$ if $d > n$. In the case where $q = 2$, it is sufficient for $d \geq n$.

Transitivity

Definition

The action is transitive on a set of polynomials S if for all $f, g \in S$, there exists a matrix $A \in \mathrm{GL}_n(\mathbb{F}_q)$ so $A \cdot f = g$. Equivalently, S is a subset of a single orbit.

Theorem

Because of the degree preserving property of the action, it is never transitive on $\mathcal{P}_{n,d,q}$ nor $\mathcal{P}^*_{n,d,q}/\sim$.

Asymptotic Transitivity

Definition

We say that the action is *asymptotically transitive* for a specific n and d if

$$\lim_{q\to\infty} \frac{\max\{|\operatorname{Orb}(f)| \ : \ f \in \mathcal{P}_{n,d,q}\}}{|\mathcal{P}_{n,d,q}|} = 1$$

Note: Asymptotic Transitivity was introduced by Cigole Thomas in her 2022 PhD thesis [2].

Theorem

The action is never asymptotically transitive... before quotienting.

Lemma

The action is transitive on $\mathcal{H}_{m,1,q}$, the set of homogeneous linear polynomials.

Theorem

The action NOT transitive but IS asymptotically so on linear projective space $\mathcal{P}^*_{1,d,q}/\sim$.

Proof (half of one anyway).

The size of the largest orbit (up to scalar equivalence) is the set of non-homogeneous linear polynomials, whose cardinality is $q^n - 1$, whereas the total set of polynomials has cardinality $q^n + \frac{q^{n-1}}{q-1}$.
The ratio of these two expressions approaches 1 as $q \to \infty$, and we get asymptotic transitivity. □

Unanswered Questions

- Compute the stabilizer group $\text{Stab}(f) = \{A \in \text{GL}_n(\mathbb{F}_q) : A \cdot f = f\}$, and have it act on the variety $V(f) = \{\vec{u} \in \mathbb{R}^n : f(\vec{u}) = 0\}$. What is the relationship between these two dynamical systems?
- We only looked at the extreme orbits (fixed orbits, transitive orbits, and full orbits). What is the entire spectra of orbits?
- Will Dr. Lawton ever be satisfied of his thirst for blood?[1]

[1] Gabe's Conjecture: No.

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References

[1] Robert Steinberg. "On Dickson's theorem on invariants". In: *Journal of the Faculty of Science. University of Tokyo. Section IA. Mathematics* 34 (1987), pp. 699–707.

[2] Cigole Thomas. *Stratification and Arithmetic Dynamics on Character Varieties*. Thesis (Ph.D.)–George Mason University. ProQuest LLC, Ann Arbor, MI, 2022, p. 117. ISBN: 979-8841-75287-5. URL: http://gateway.proquest.com/openurl?url_ver=Z39.88-2004&rft_val_fmt=info:ofi/fmt:kev:mtx:dissertation&res_dat=xri:pqm&rft_dat=xri:pqdiss:29169430.