Geometry Through Crochet

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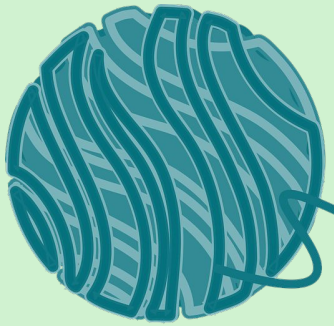
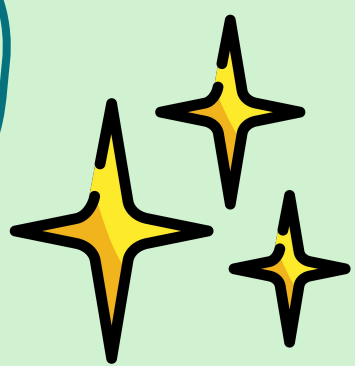
Spring 2023





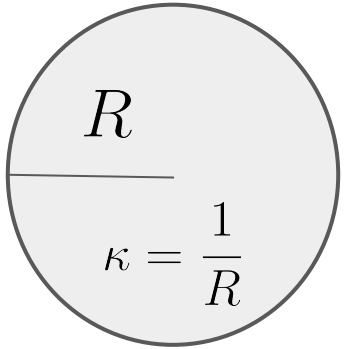
Curvature

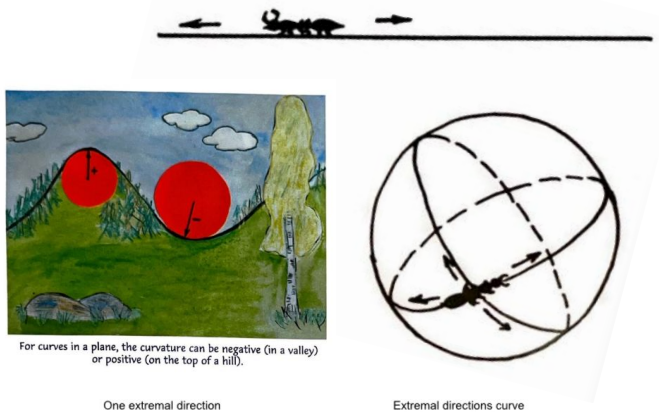
In R^2:

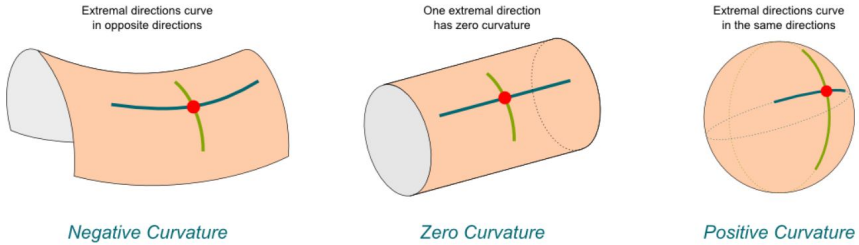
- Lines have **zero curvature**
- The curvature of a circle is **inversely proportional** to its radius

In R^3:

- Cylinders have **zero curvature**
- A sphere has **constant positive curvature, equal to** $\frac{1}{R^2}$

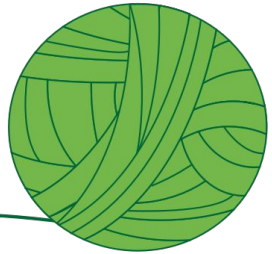




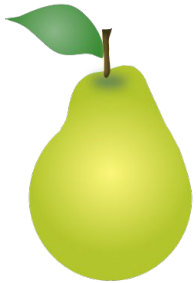


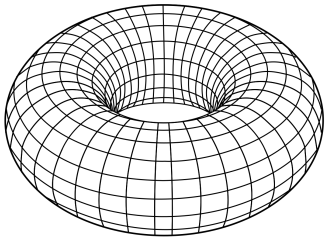
Curvature in various objects

Positive



Mixed

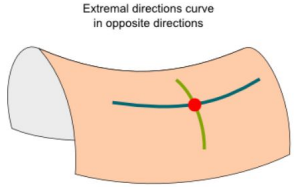




Negative

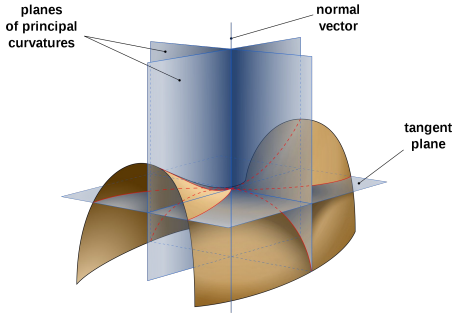
Hyperbolic Plane and visualizations

- The plane has constant negative curvature and extends indefinitely in R^3, unlike the sphere
- Henri Poincare responsible for two classical models of the plane:
  - Poincaré disk
  - Upper half plane



Negative Curvature

Hyperbolic plane made with paper annuli.



H. S. M. Coxeter's tiling of Poincaré disk.

Why Crochet?

Crochet allows us to increase the stitches in an easy and controlled manner.

Hyperbolic plane made with paper annuli.

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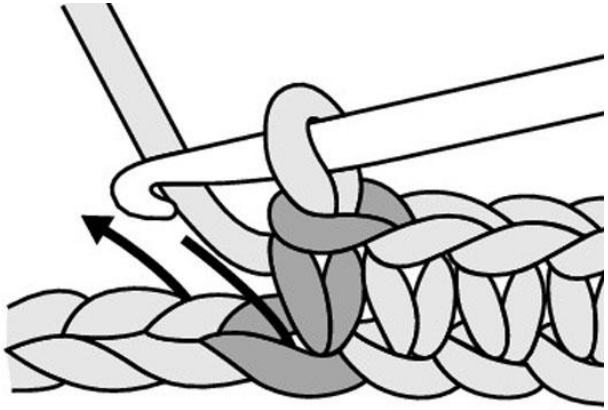
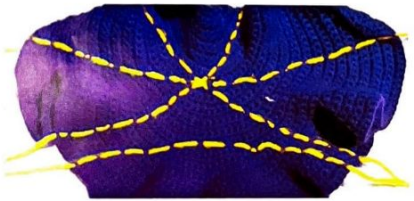


My first model of the hyperbolic plane made in 1997.

Flattening negative curvature.



Lines through the same point and not intersecting another line in the hyperbolic plane.

Discrete Representations

- Tiling of regular polygons that approximates a sphere, the hyperbolic plane, or other objects in R^3
- D4 and D20 for nerds

Seven hexagons around a heptagon approximates a hyperbolic plane (constant negative curvature).

Crocheted Representations

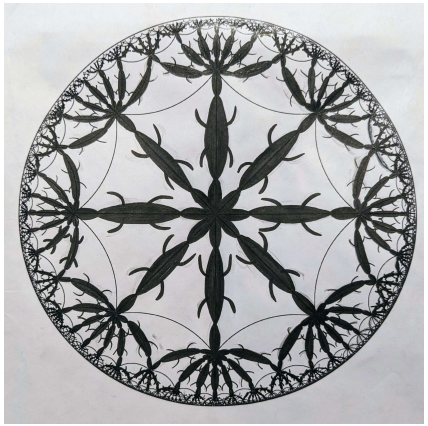
What did you think we did all semester????

Joint Hyperbolic Plane

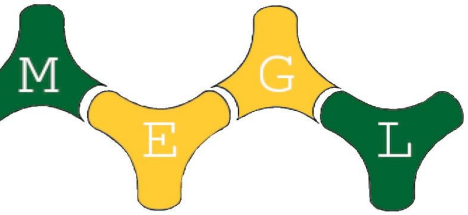
- Varying negative curvature
- Future plan: Approximate curvature with discs and/or spheres of positive curvature

Hyperbolic Tiling

- Poincaré disc-inspired hyperbolic tiling.
- Future plans: Constructing a Poincaré disc with Mobius Transformations



Collaboration



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Future Work

- Constructing the Poincare Disk with mobius transformations
- Exploring other crochet surfaces, like the lochness monster surface.

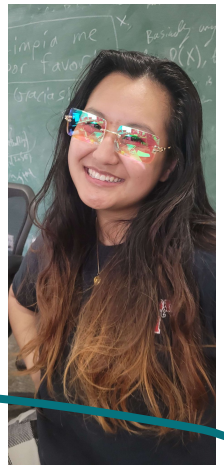
Credit to Savannah Crawford

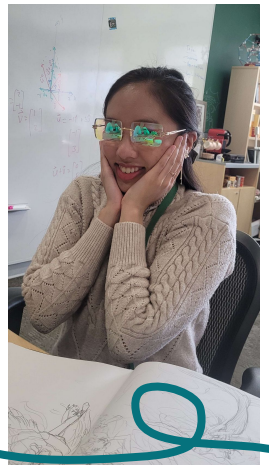
- Further collaboration with the UVA

Acknowledgements

It has been a pleasure to have this project organized and managed by both Madeline as our graduate advisor, and Professor Bray as our faculty advisor! Extra special thanks to both of them for providing lab members with materials to crochet with as well!

THANK YOU!!!!





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Ideal Sphere Pdf

Hyperbolic Flower Pdf