The Effects of Surface Tension on Floating 3D-Printed Objects

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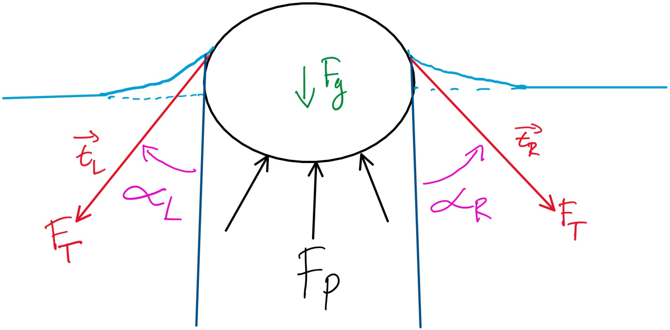
Introduction

In this study, we investigate the behavior of systems involving surface tension, gravitational, and pressure/buoyant forces. By analyzing the interplay of these forces and the phenomenon of Marangoni propulsion, we aim to gain a deeper understanding of the dynamics of floating objects and their movement on liquid surfaces.

Force Balance Equation

Consider a rigid stationary floating object, we characterize its orientation with the balance of gravitational force, pressure/buoyant force, and surface tension force:

$$\vec{F_g} + \vec{F_p} + \vec{F_T} = 0$$



Gravitational and Pressure/Buoyant Forces

Gravitational Force

The gravitational force on the object is given by:

$$\vec{F_g} = M_{obj}\vec{g}$$

Pressure/Buoyant Force

Let $\hat{n}_{obj}$ = unit normal force directed out of object, $\partial\Omega_{sub}$ = the boundary submerged region of object, and p_A = atmospheric pressure. The pressure force on the object is:

$$\vec{F_p} = -\ell \int_{\partial\Omega_{sub}} (p(s) - p_A)\hat{n}_{obj}(s)ds$$

Surface Tension Force

The surface tension force is in the direction tangent to the meniscus at the point where the object intersects the surface of water.

$$\vec{F_T} = \ell \gamma_L \vec{t_L} + \ell \gamma_R \vec{t_R}$$

We can express this force in vertical and horizontal components:

$$\vec{F}_T^y = \ell(-\gamma_L \cos\alpha_L - \gamma_R \cos\alpha_R) \tag{1}$$

$$\vec{F}_T^x = \ell(-\gamma_L \sin\alpha_L + \gamma_R \sin\alpha_R) \tag{2}$$

Archimedes Principle and the Force of Surface Tension

Archimedes Principle

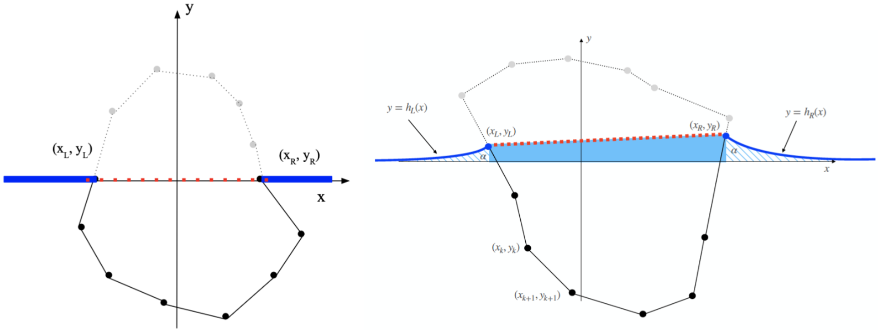
The buoyant force acting on a floating object is opposite and equivalent to the force of gravity acting upon the object, or in other words:

$$M_{obj}g = \rho_f V_{sub} g.$$

That is, the pressure force in the vertical direction is the weight of the displaced fluid.

When the force of surface tension is accounted for in the force balance equation, a revised interpretation of displaced fluid is necessary.

A Floating Object without and with Surface Tension



a. No Surface Tension b. Surface Tension

Forces on a Floating Object Experiencing Surface Tension

Surface Tension Force Derivation

Using the equations for the force of surface tension and the pressure force,

$$\begin{aligned}
\vec{F}_T^y &= -\ell\rho_f g \left\{ \int_{-\infty}^{x_L} h_L dx + \int_{x_R}^{\infty} h_R dx \right\} \\
\vec{F}_T^x &= \ell \left\{ \frac{\rho_f g}{2}({y_L}^2 - {y_R}^2) + (\gamma_R - \gamma_L) \right\}
\end{aligned}$$

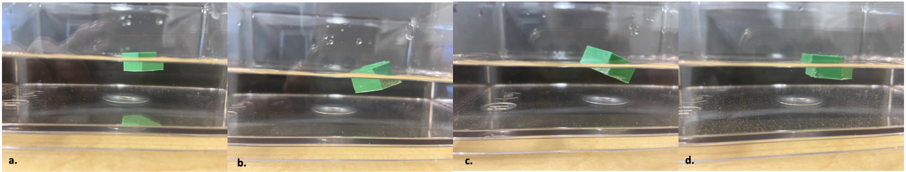
Pressure Force Derivation

$$\vec{F}_p = -\ell \int_{\partial\Omega_{sub}} (p(s) - p_A)\hat{n}_{obj}(s)ds$$

is the expression for the force of water pressure on the object, which is derived from

$$\vec{F}_p = \ell\rho_f g \left\{ \frac{1}{2}({y_R}^2 - {y_L}^2), A_{sub} - A_{\text{meniscus}} \right\}$$

By changing the physical properties of the object, contact angle at the object also changes. This effect is demonstrated in the figures below.



3D Printed "boats" which have been sprayed with water-wicking substance in four manners. From left to right the boats have: **a.** no spray, **b.** spray on only the back three faces, **c.** spray on only the front two faces, and **d.** spray on only the bottom face.

Marangoni Propulsion

Definition (Marangoni Propulsion)

Locomotion in or on a liquid caused by a change in the surface tension on one end of an object. This yields two different γ values, γ_L and γ_R.

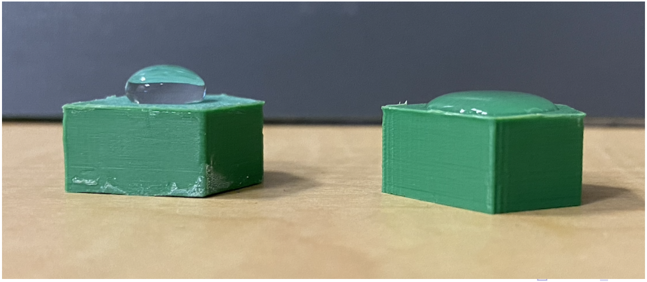
When surface tension constant $\gamma_L \neq \gamma_R$, the horizontal balance of forces affecting an object becomes non-zero. As a result, the object experiences a horizontal propulsive force, called Marangoni propulsion, given by

$$F_{prop} = \ell(\gamma_R - \gamma_L)$$

Future Work

We plan to continue exploring the effects of surface tension on floating objects in systems such as the following:

- extend our work from static to dynamical systems
- explore the role of surface tension in bio-locomotion
- floating entomological phenomena, such as fire ant rafts



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