The Effects of Surface Tension on Floating 3D-Printed Objects

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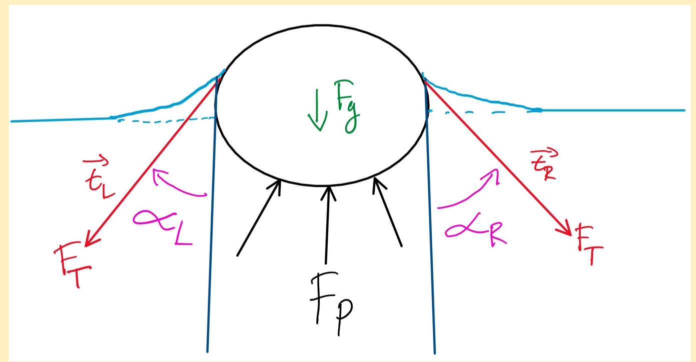
Introduction

In this study, we investigate the behavior of systems involving surface tension, gravitational, and pressure/buoyant forces, specifically looking at Marangoni propulsion on 3D-printed pentagonal boats. By analyzing the interplay of these forces and the phenomenon of Marangoni propulsion, we aim to gain a deeper understanding of the dynamics of floating objects and their movement on liquid surfaces.

Relevant Forces



Force-Balance Equation

Consider a rigid stationary floating object, we characterize its orientation with the balance of gravitational force, pressure/buoyant force, and surface tension force:

$$\vec{F_g} + \vec{F_p} + \vec{F_T} = 0$$

Gravitational Force

The gravitational force on the object is given by:

$$\vec{F_g} = M_{obj}\vec{g}$$

where $\vec{g}$ is the acceleration due to gravity

Pressure/Buoyant Force

Let $\hat{n}_{obj}$ = unit normal force directed out of object, $\partial\Omega_{sub}$ = the boundary submerged region of object, and p_A = atmospheric pressure. The pressure force on the object is:

$$\vec{F_p} = -\ell \int_{\partial\Omega_{sub}} (p(s) - p_A)\hat{n}_{obj}(s)ds$$

Surface Tension Force

The surface tension force is in the direction tangent to the meniscus at the point where the object intersects the surface of water.

$$\vec{F_T} = \ell\gamma_L\vec{t_L} + \ell\gamma_R\vec{t_R}$$

We can express this force in vertical and horizontal components:

$$\vec{F}_T^y = \ell(-\gamma_L\cos\alpha_L - \gamma_R\cos\alpha_R) \quad (1)$$

$$\vec{F}_T^x = \ell(-\gamma_L\sin\alpha_L + \gamma_R\sin\alpha_R) \quad (2)$$

Surface Tension on Floating 3D-Printed Objects

Archimedes Principle without Surface Tension

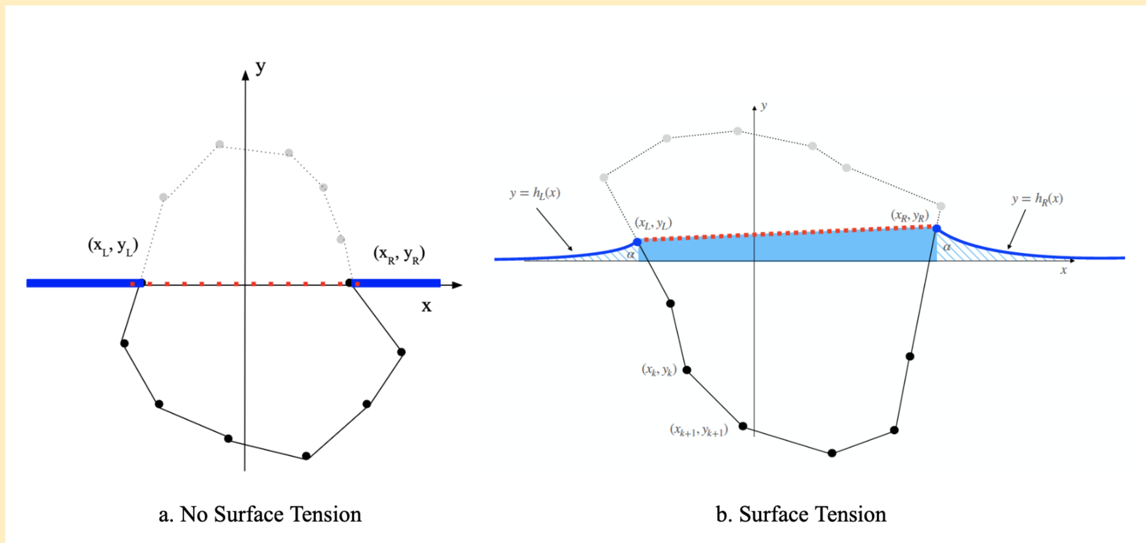
The buoyant force acting on a floating object is opposite and equivalent to the force of gravity acting upon the object, or in other words:

$$M_{obj}g = \rho_f V_{sub}g.$$

That is, the pressure force in the vertical direction is the weight of the displaced fluid. See diagram a.

Surface Tension Term in Archimedean Force Balance Equation

When the force of surface tension is accounted for in the force balance equation, a revised interpretation of displaced fluid is necessary. See diagram b.



Surface Tension Force Derivation

Using equation 1 and 2

$$\vec{F}_T^y = -\ell\rho_f g \left\{ \int_{-\infty}^{x_L} h_L dx + \int_{x_R}^{\infty} h_R dx \right\}$$

$$\vec{F}_T^x = \ell \left\{ \frac{\rho_f g}{2}({y_L}^2 - {y_R}^2) + (\gamma_R - \gamma_L) \right\}$$

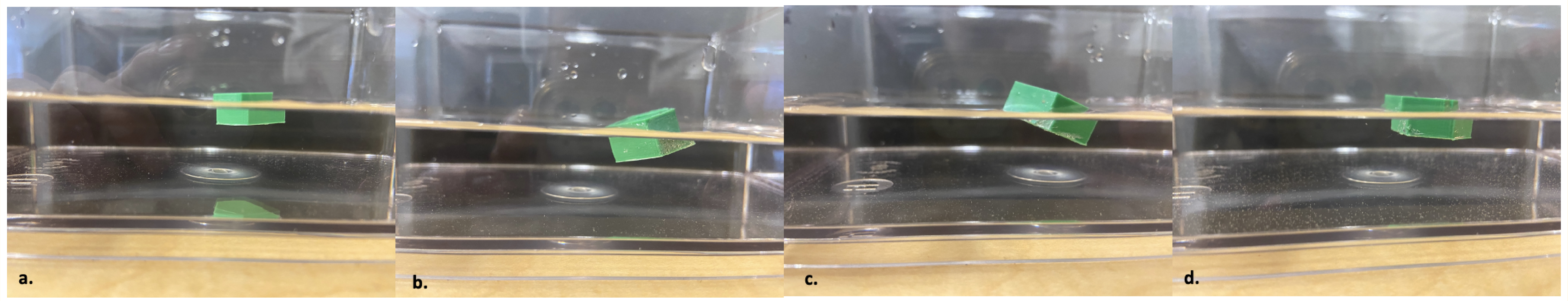
Pressure Force Derivation

$$\vec{F_p} = -\ell \int_{\partial\Omega_{sub}} (p(s) - p_A)\hat{n}_{obj}(s)ds$$

is the expression for the force of water pressure on the object, which is derived from

$$\vec{F_p} = \ell\rho_f g \left\{ \frac{1}{2}({y_R}^2 - {y_L}^2), A_{sub} - A_{\text{meniscus}} \right\}$$

By changing the physical properties of the object, contact angle at the object also changes. This effect is demonstrated in the figures below.



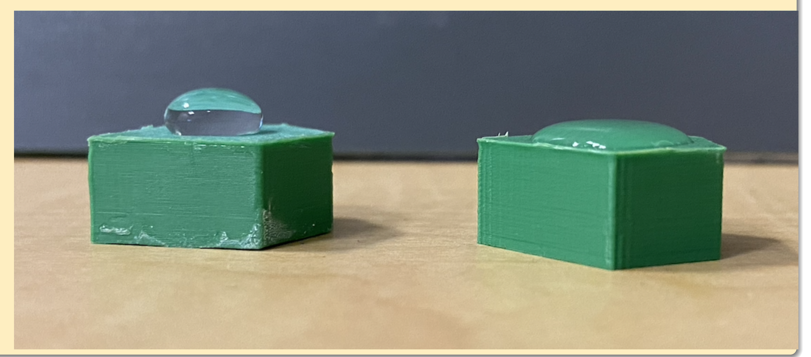
3D Printed "boats" which have been sprayed with water-wicking substance in four manners. From left to right the boats have: **a.** no spray, **b.** spray on only the back three faces, **c.** spray on only the front two faces, and **d.** spray on only the bottom face.

When surface tension constant $\gamma_L \neq \gamma_R$, the horizontal balance of forces affecting an object becomes non-zero. As a result, the object experiences a horizontal propulsive force, called Marangoni propulsion, given by

$$F_{prop} = \ell(\gamma_R - \gamma_L)$$

Definition (Marangoni Propulsion)

Locomotion in or on a liquid caused by a change in the surface tension on one end of an object. This yields two different γ values, γ_L and γ_R.



Future Work

We plan to continue exploring the Marangoni phenomenon and its applications, particularly in its ability to model more complicated biological systems like a fire ant raft. Our goal is to extend our work from the static setting to a dynamical setting, where we can model the behavior of the system as it evolves over time. This will allow us to better understand the complex interactions between various components of the system and to identify opportunities for further research and innovation.

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