Tipping in Climate Impact Models

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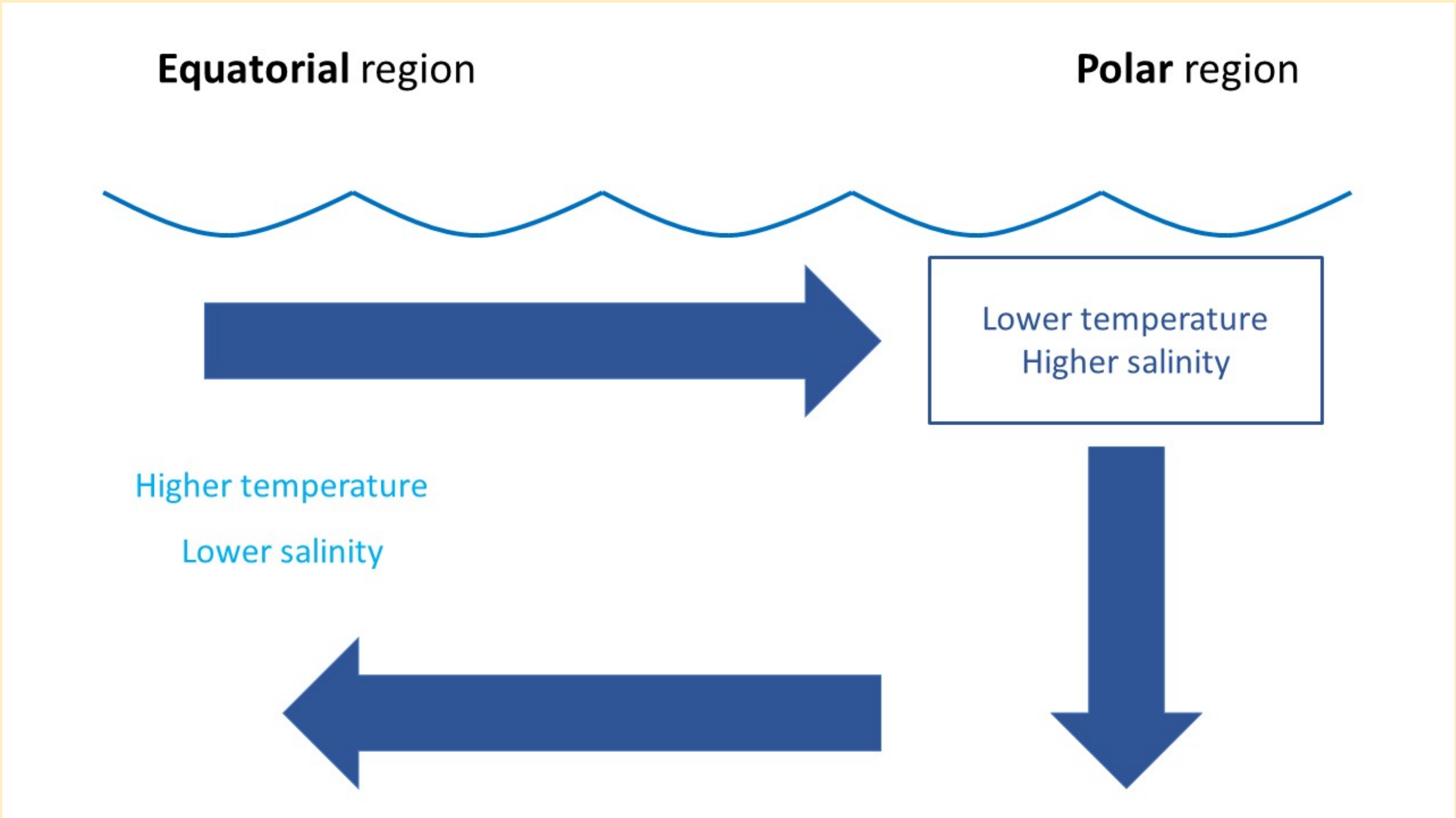
Thermohaline Circulation



Thermohaline circulation is the global circulation of the world's ocean waters, driven by density gradients produced by variations in temperature and salinity.

Bifurcation Tipping

B-tipping tipping is a sudden transition that is induced by slow changes to a parameter of the underlying dynamical system which can reduce the stability and ultimately destroy a stable node. This sudden transition occurs at a critical level. Fixing this parameter value would generate a different phase portrait each time.

Noise-Induced Tipping

N-tipping or noise-induced tipping can be characterized by a ball in two valleys on a potential graph, the "noise" (random perturbations) or "Stochastic Process" will drive one ball to the next valley. The depth of the valley will determine how much noise is required to shift the ball to the next valley. This tipping does not alter the landscape of the potential graph.

Rate-Induced Tipping

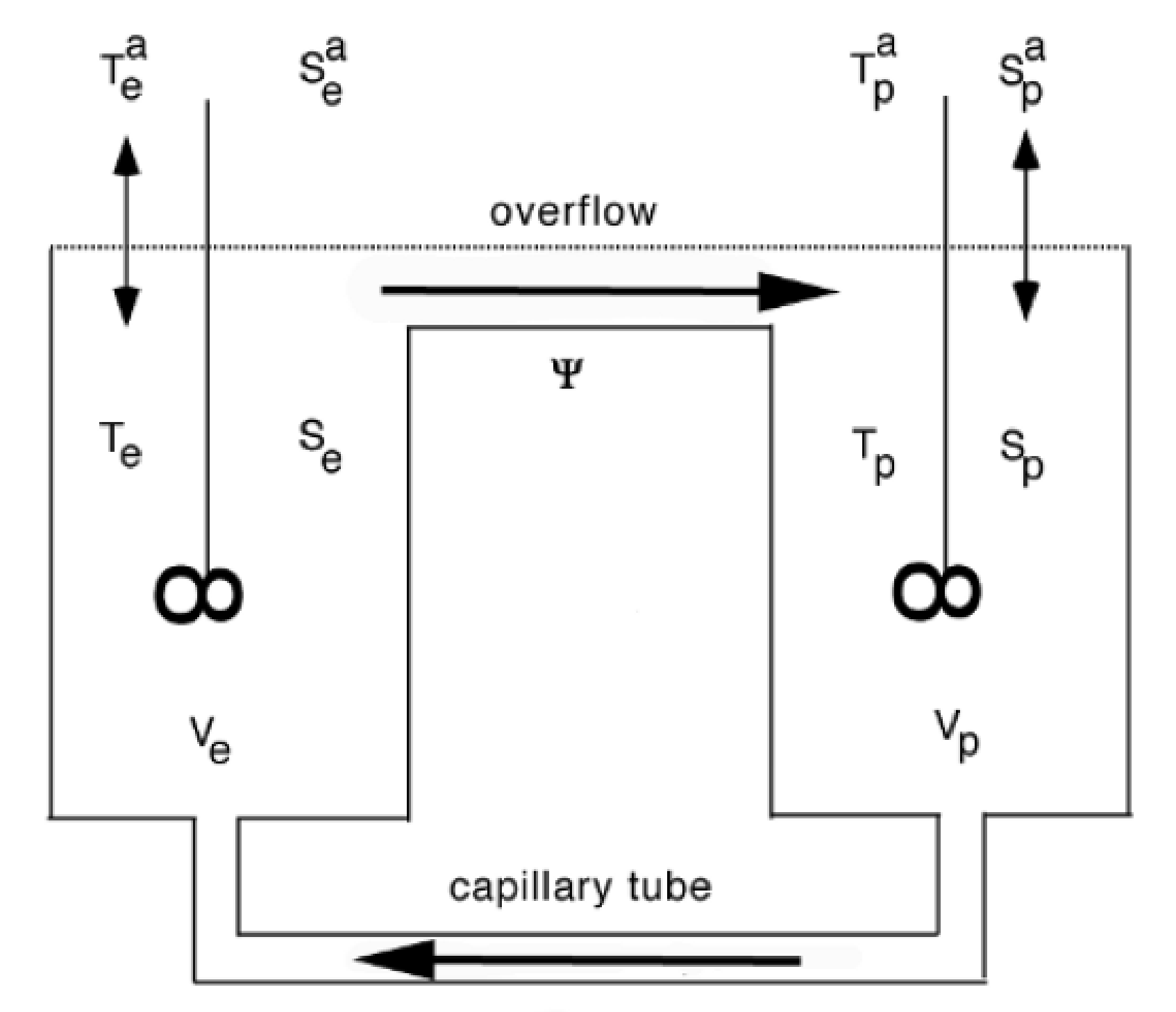
R-tipping occurs when time variation of input parameters of a dynamical system interacts with system timescales to give genuine non-autonomous instabilities. There is no critical level, but rather a critical rate at which the system will tip. It does not depend on parameter and uses a ramp function with a critical threshold where the tipping from one stable state to another usually occurs.

Importance of Tipping

Tipping is important in our dynamical system because we can examine real world data/parameters which can have a direct impact to Earth's climate, such as a change in direction of the ocean current's thermohaline circulation. The tipping can alter the Earth's climate.

The Stommel System

The Stommel Model



Theorem

Bendixson-Dulac Theorem C^1 Dulac function ($\varphi(T,S)$)
Such that the expression:

$$\frac{\partial(\varphi f)}{\partial T}+\frac{\partial(\varphi g)}{\partial S}$$

Has the same sign everywhere ($\neq 0$)
Then the nonautonomous planar system has no periodic solution.

Stommel Proof

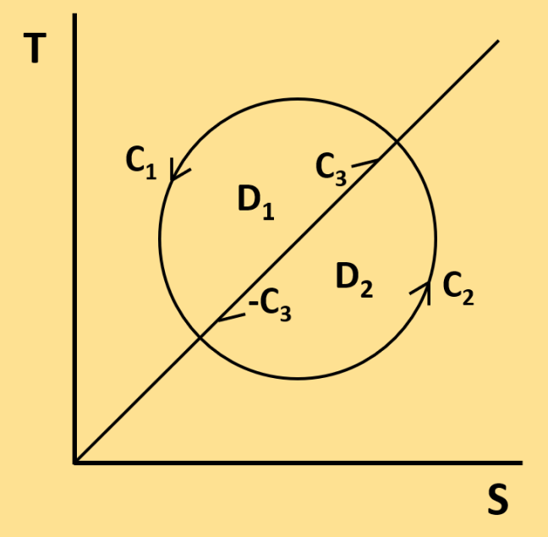


Figure: Schematic for periodic orbit for $T,S \geq 0$.

Nondimensional equations

$$\frac{dT}{dt}=\eta_1-T(1+|T-S|)$$

$$\frac{dS}{dt}=\eta_2-S(\eta_3+|T-S|)$$

Equilibrium solutions

$$T=\frac{\eta_1}{1+|\Psi|} \quad \text{and} \quad S=\frac{\eta_2}{\eta_3+|\Psi|}$$

Steady States

TH regime

$T>S$, $|\Psi|=\Psi$,

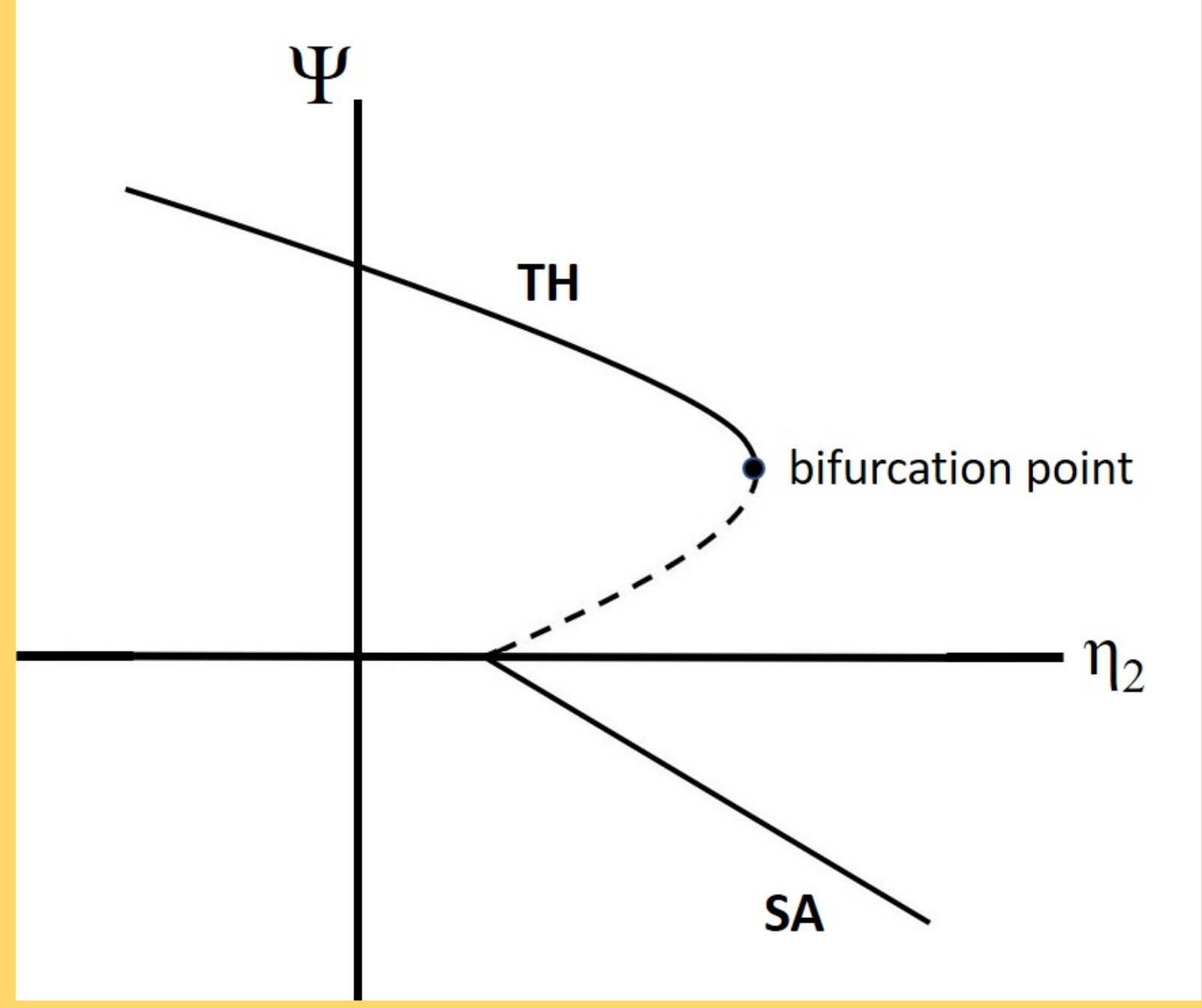
$$\eta_2=-\Psi^2-\eta_3\Psi+\eta_1\left(\frac{\eta_3+\Psi}{1+\Psi}\right)$$

SA regime

$S>T$, $|\Psi|=-\Psi$

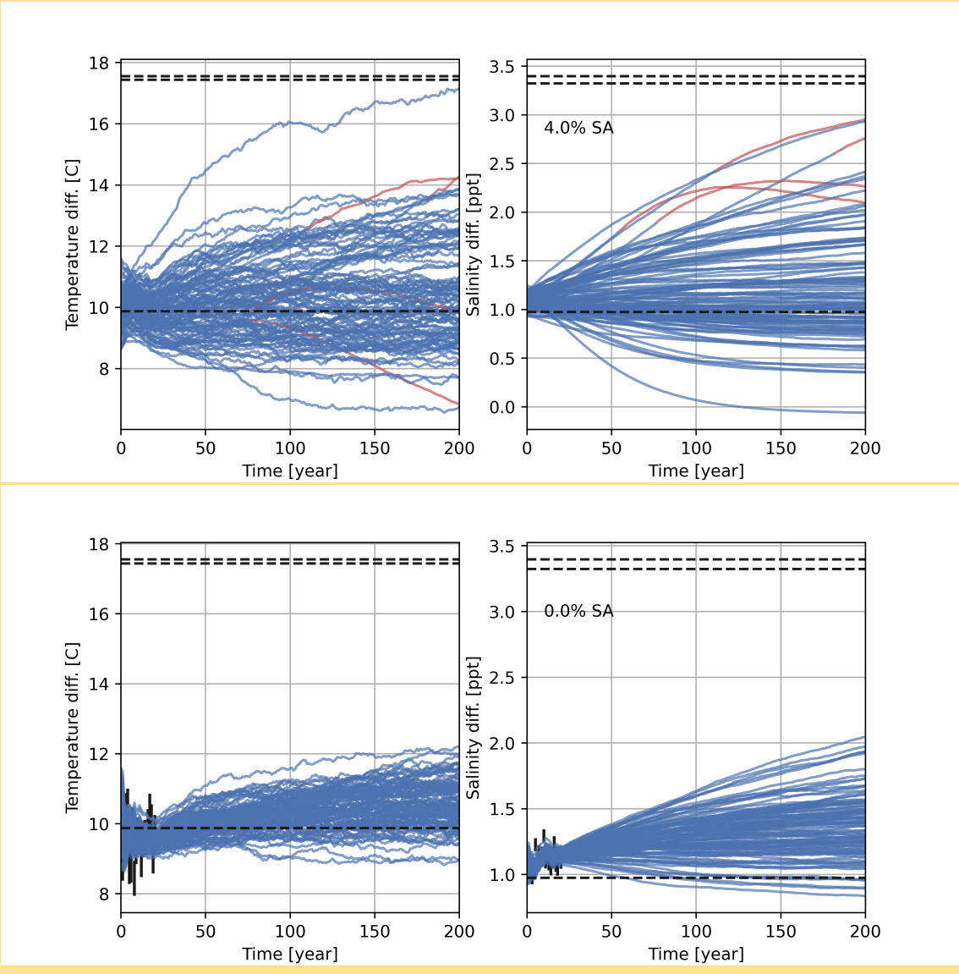
$$\eta_2=\Psi^2-\eta_3\Psi+\eta_1\left(\frac{\eta_3-\Psi}{1-\Psi}\right)$$

Bifurcation Diagram

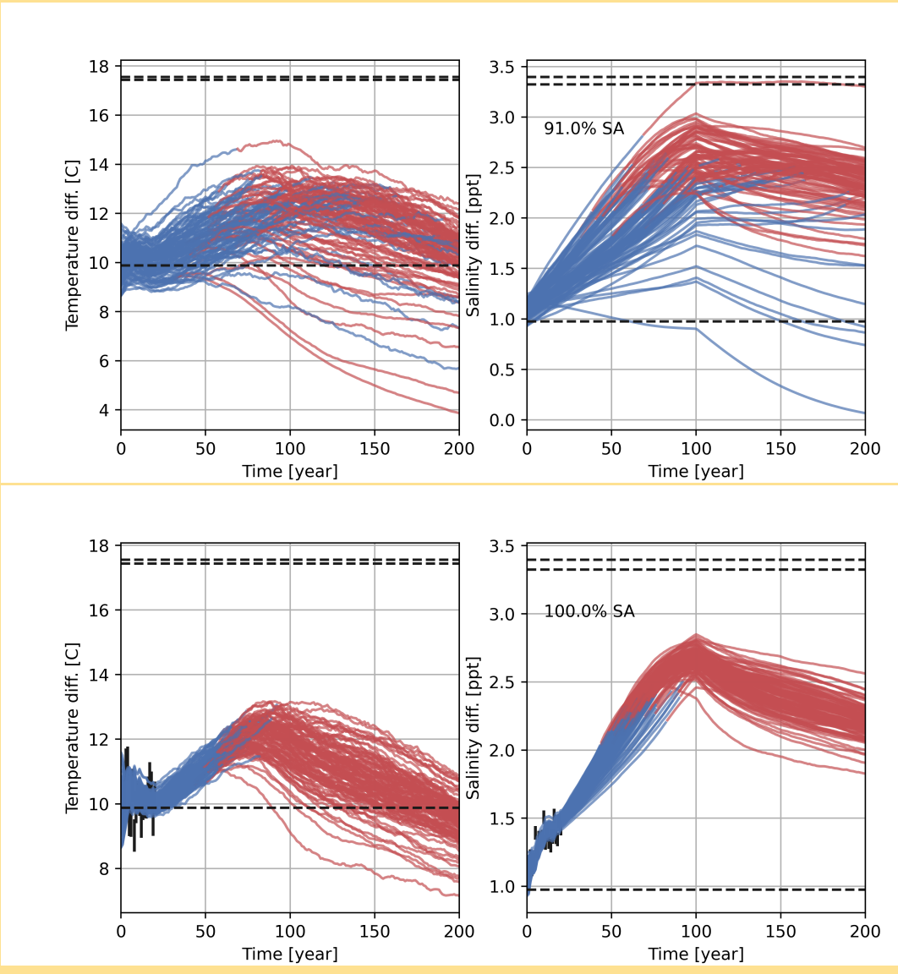


Graphics

Reference Ensemble Run



Ensemble Runs With Effect of Climate Change



Data Assimilation

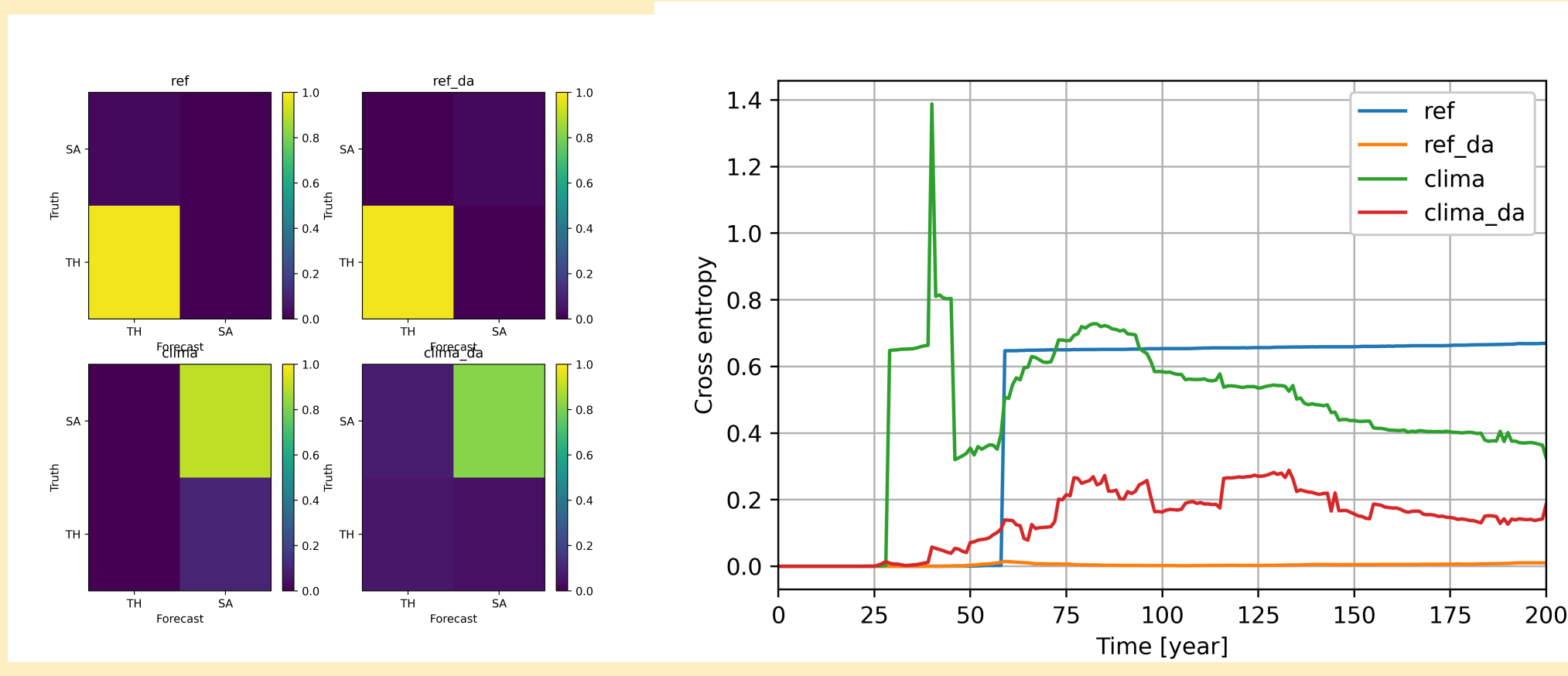
Data assimilation is a set of techniques in which real world observations are used to make sequential, statistical inferences about the state of a system, given a dynamical model and its static parameters. In other words, it is a way to fuse data and models together to make more accurate predictions about the true state of a system at some point in time.

Ensemble Kalman Filter (EnKF)

The EnKF is a Monte-Carlo estimation of the Kalman filter which helps us to deal with large, nonlinear systems. The Kalman Filter alone only works on linear systems, and models uncertainty as a covariance matrix between two Gaussian distributions (the prediction and observation). For high-dimensional systems, maintaining this covariance matrix is computationally prohibitive. The EnKF solves this problem by using a Monte Carlo approach to estimate the mean and covariance. This is much easier computationally, as well as having the benefits of being non-invasive (the model is treated as a black box), not requiring linearization of the model, and being highly parallelizable.

Conclusions/Future Work

In our simulations, we have confirmed our fears that the melting of the Greenland ice sheet and north pole could indeed potentially lower salinity near the north pole to such extent that the regime changes from TH to SA. We do note that however, assimilating data from more years of observation, more specifically, when the years of observations are greater than the time of ice cap melting, the spread of the ensemble members over time is reduced significantly. Thus, in order for our prediction to be even more accurate, we note that our model should be continually updated as more data becomes available.



(a) (b)

Figure: In (a) we have a confusion matrix for our 4 different scenarios where in some scenarios, temperature and salinity are assimilated for 20 years. In (b), we present a cross entropy plot for our 4 different scenarios.

References

[1] H.A. Dijkstra. *Dynamical Oceanography*. Springer Berlin Heidelberg, 2008.