The Stability of Floating Objects

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Abstract

We investigate a variety of floating shapes with two-dimensional cross sections and identify analytically and/or computationally a potential energy landscape that helps identify stable and unstable floating orientations. We compare our analyses and computations to experiments on floating objects to deduce the number of orientations and their evolution through shifting the center of mass.

Center of Gravity

$$\vec{G} = \frac{1}{M_{obj}} \int_{\Omega} \vec{x}\rho(\vec{x})dV$$

Center of Buoyancy

Center of mass of the displaced fluid

$$\vec{B} = \frac{1}{M_{sub}} \int_{\Omega_{sub}} \vec{x}\rho_{fluid}dV$$

Archimedes' Principle

The upward buoyant force exerted on an object wholly or partially submerged, is equal to the weight of the displaced fluid.

$$M_{obj}g = \rho_{fluid}V_{sub}g \qquad \frac{V_{sub}}{V_{obj}} = \frac{\rho_{obj}}{\rho_{fluid}}$$

Algorithm: Computing Potential Energy Landscapes

- Given a set of boundary points
- Compute $\vec{G}$ (center of mass)
- For $\theta \in [0, 2\pi]$ (orientation of object)
  - Identify water line consistent with Archimedes' Principle
  - Compute $\vec{B}(\theta)$ (center of buoyancy)
  - Potential Energy

$$U(\theta) \sim \hat{n}(\theta) \cdot (\vec{G} - \vec{B}(\theta))$$

Experiments and Data Acquisition

Designed and created samples through OpenSCAD and 3D printing. We compared our experiments of 3D printed floating objects to both computational and analytical results.

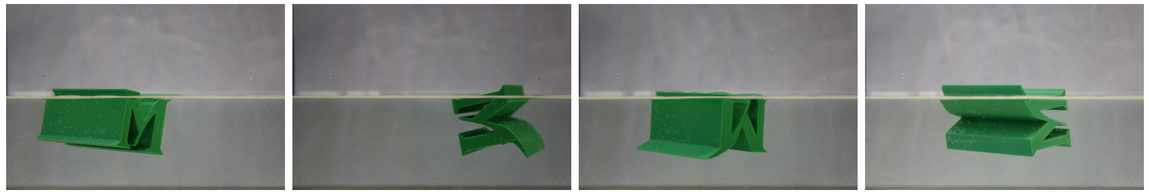

Figure: The four stable orientations of the floating Mason M.

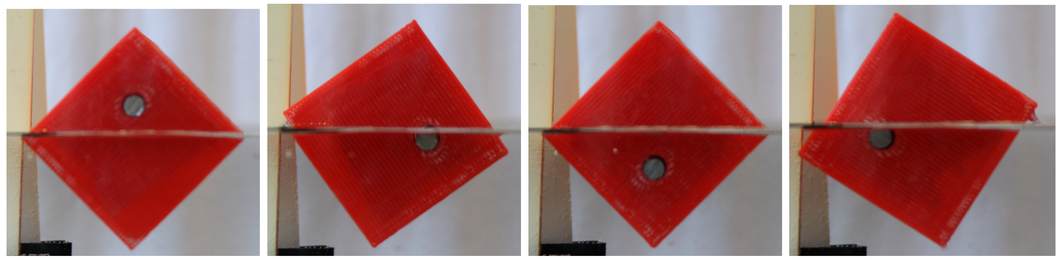

Figure: Four floating orientations for an object with center of gravity off center

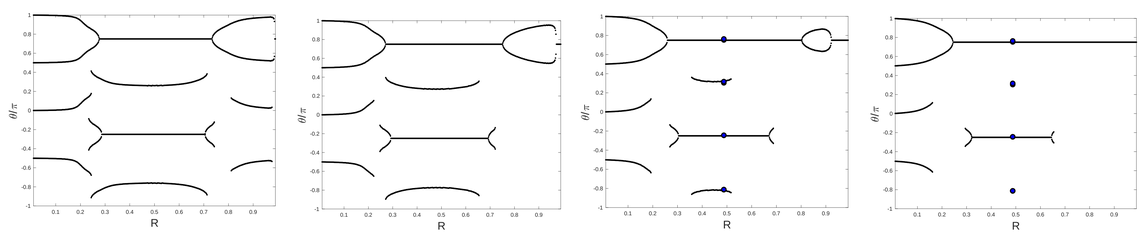

Figure: Stable equilibrium angles versus density ratio R for off-center squares.

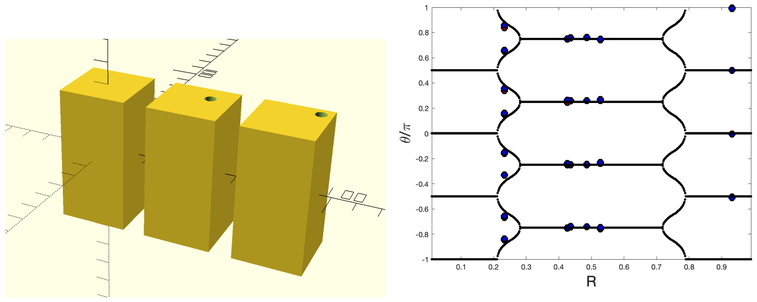

Figure: OpenSCAD file and equilibrium angles vs density ratio for center of gravity for uniform density square

Results

It is found that a floating body is in stable equilibrium if its center of gravity has a minimum height with respect to its related center of buoyancy. Using 3D printed models we were able to produce and collect experimental results in agreement with the theory.

Potential Energy Landscape

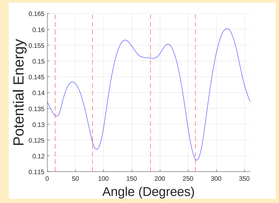

Figure: The Mason M

- Red dotted lines are experimental stable orientations ($\sim$20°,85°,185°,260°)
- Theoretical predicted orientations at minima of potential energy function

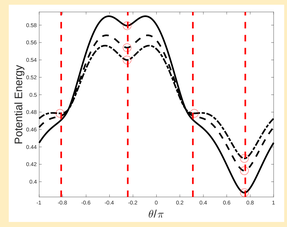

Figure: Off-center Square

- Red dotted lines are experimental stable orientations
- Solid black curve is the energy function for best estimate of center of gravity
- Dashed black curves are energy functions for nearby centers of gravity

Open problems

Ulam asks "Is a solid of uniform density which will float in water in every position a sphere?" We would like to investigate the stability of Reuleaux shapes in 3D. We would also like to develop analytical results for rectangles.

Acknowledgements

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References

Daniel M. Anderson, Brandon G. Barreto-Rosa, Joshua D. Calvano, Lujain Nsair, and Evelyn Sander "Mathematics of Floating 3D Printed Objects." submitted for publication 2022.