The Stability of Floating Objects

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December 3, 2021

Questions

- Can you identify equilibrium orientations?
- Can you identify stable equilibrium orientations?

Center of Gravity

- Discrete Sum

$$M_{tot}\vec{G} = \sum_{i=1}^{n} m_i \vec{x}_i$$

- Continuous Sum

$$\vec{G} = \frac{1}{M_{obj}} \int_{\Omega} \vec{x}\rho(\vec{x})dV$$

Center of Buoyancy

- Center of mass of the displaced fluid

$$\vec{B} = \frac{1}{M_{sub}} \int_{\Omega_{sub}} \vec{x}\rho_{fluid}dV$$

Archimedes' Principle

- The upward buoyant force exerted on an object wholly or partially submerged, is equal to the weight of the displaced fluid.

$$M_{obj}g = \rho_{fluid}V_{sub}g$$
$$\frac{V_{sub}}{V_{obj}} = \frac{\rho_{obj}}{\rho_{fluid}}$$

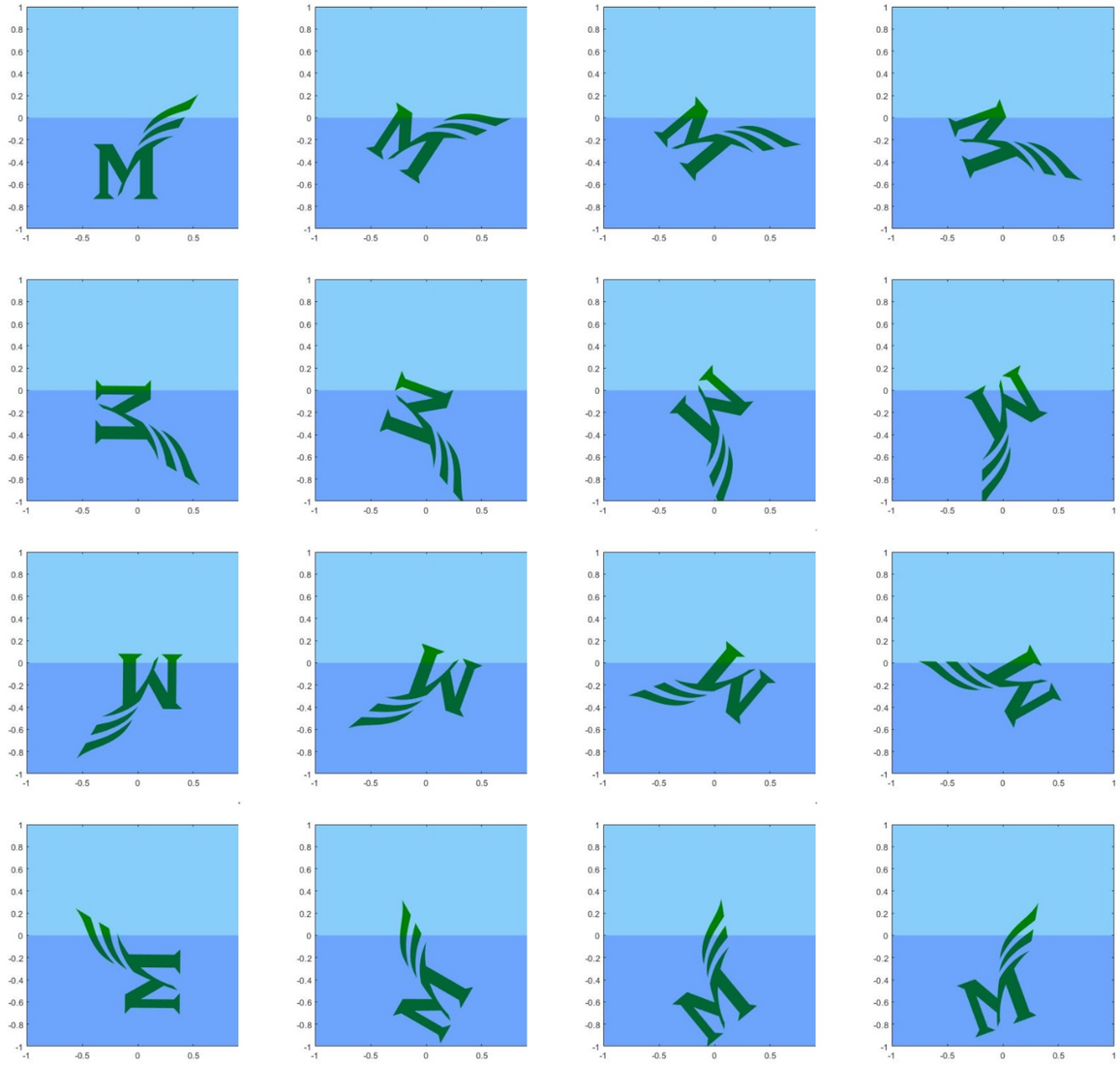
- For an iceberg in seawater

$$\frac{\rho_{obj}}{\rho_{fluid}} \approx 0.9$$

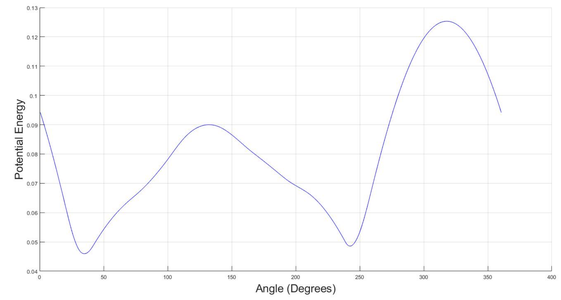
Algorithm: Computing Potential Energy Landscapes

- Given a set of boundary points
- Compute $\vec{G}$ (center of mass)
- For $\theta \in [0, 2\pi]$ (orientation of object)
  - Identify water line consistent with Archimedes'
  - Compute $\vec{B}(\theta)$ (center of buoyancy)
  - Potential Energy

$$U(\theta) \sim \hat{n}(\theta) \cdot (\vec{G} - \vec{B}(\theta))$$



Potential Energy Landscape



Answers

Most of the examples in the 4x4 grid are not equilibrium orientations, although they all do satisfy Archimedes' Principle.

- Stable Equilibrium Positions: at $\theta = 33.5$ and $\theta = 240.5$
- Unstable Equilibrium Positions: $\theta = 131.5$ and $\theta = 320.5$

Summary

An equilibrium orientation of a floating body occurs when the center of gravity (the center of mass of the whole object), and the center of buoyancy (the center of mass of just the submerged part), are vertically aligned. The potential energy of a system in stable equilibrium has a minimum value. This property is used to derive a formula that is useful in the determination of stability of a floating body. It is found that a floating body is in stable equilibrium if its center of gravity has a minimum height with respect to its related center of buoyancy.

More Questions

- Relationships between aspect ratios and the number of favorable orientations.
- Concavity, vertices, and invariant geometries.

Acknowledgements

We would like to thank our mentor and advisor, Prof. Anderson, and MEGL for supporting our research.

References

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