The Stability of Floating Objects

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December 3, 2021

Overview

- Objective: Understanding orientations of static floating objects.
  - Consequences of evolution and shape deformation on orientation
  - Melting and Calving
  - Explaining observed patterns
- Preliminaries : Center of Mass, Center of Buoyancy, Archimedes Principle

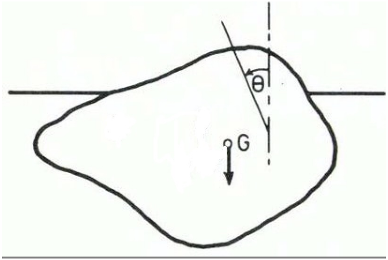
Center of Mass, $\vec{G}$

- Discrete Sum

$$M_{tot}\vec{G} = \sum_{i=1}^{n} m_i \vec{x}_i$$

- Continuous Sum

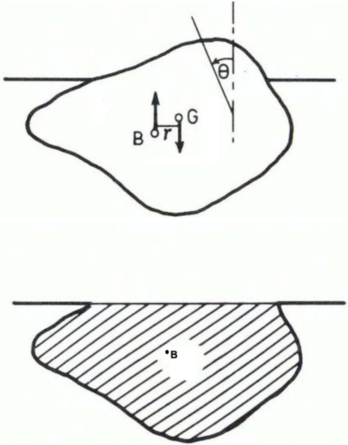
$$\vec{G} = \frac{1}{M_{obj}} \int_{\Omega} \vec{x}\rho(\vec{x})dV$$



Center of Buoyancy, $\vec{B}$

- Center of mass of the displaced fluid

$$\vec{B} = \frac{1}{M_{sub}} \int_{\Omega_{sub}} \vec{x} \rho_{fluid} dV$$



Archimedes' Principle

- The upward buoyant force exerted on an object wholly or partially submerged is equal to the weight of the displaced fluid

$$M_{obj} g = \rho_{fluid} V_{sub} g$$

$$\frac{V_{sub}}{V_{obj}} = \frac{\rho_{obj}}{\rho_{fluid}} = R$$

- For an iceberg in seawater

$$R = \frac{\rho_{obj}}{\rho_{fluid}} \approx 0.89$$

Algorithm: Compute Potential Energy Landscapes

- Given some shape
- Compute $\vec{G}$ (center of mass)
- For $\theta \in [0, 2\pi]$ (orientation of object)
  - Identify water line consistent with Archimedes'
  - Compute $\vec{B}(\theta)$ (center of buoyancy)
  - Potential Energy

$$U(\theta) \sim \hat{n}(\theta) \cdot (\vec{G} - \vec{B}(\theta))$$

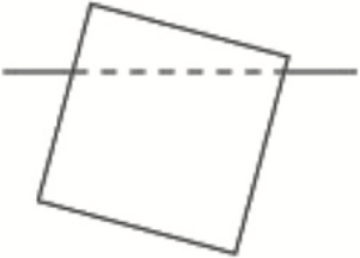
Theory for a Floating Square Pt.1 (Reid 1963 ; Feigel and Fuzailov 2021)

- When waterline intersects opposite sides:

$$\vec{B}(\theta) = \frac{1}{2(1+H)}\left\{\frac{2}{3}\tan\theta, -1 + H^2 + \frac{1}{3}\tan^2\theta\right\},$$

where $H = 2R - 1$ and θ can take on any value as long as

$$-1 + |H| \leq \tan\theta \leq 1 - |H|.$$



Theory for a Floating Square Pt.2 (Reid 1963 ; Feigel and Fuzailov 2021)

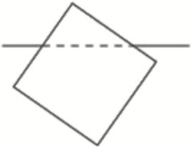
- When waterline intersects adjacent sides:

$$A_{sub}\vec{B}(\theta) = \Big\{1 - \frac{1}{2}(y_L + 1) - \frac{1}{6}(x_R^3 + 1)\tan\theta, \\ -1 + \frac{1}{2}(1 - x_R) + \frac{(1 - y_L^3)}{6\tan\theta}\Big\}.$$

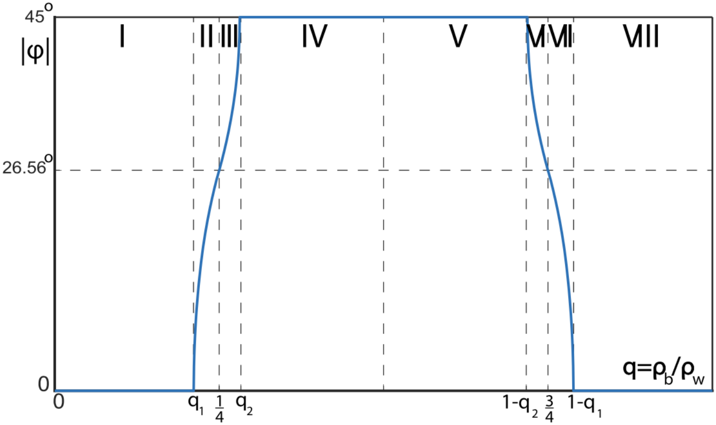
Where $A_{sub} = 4 - \frac{1}{2}(1 + x_R)(1 - y_L)$,

$$y_L = -\tan\theta(1 + x_R) + 1, \quad (1 + x_R)^2 = \frac{8 - 8R}{\tan\theta},$$

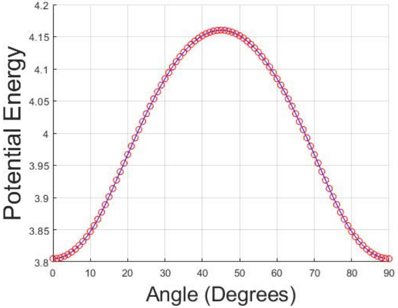
and $2 - 2R \leq \tan\theta \leq \frac{1}{2-2R}$

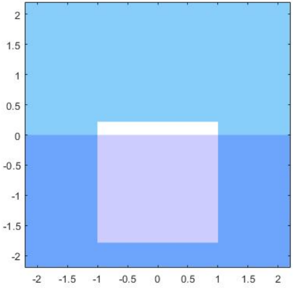


Motivation for Changing R by Feigel and Fuzailov 2021



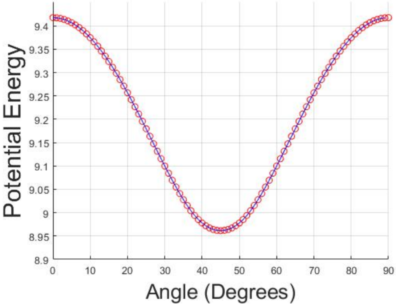
Potential Energy Landscape for Square when R=0.8912

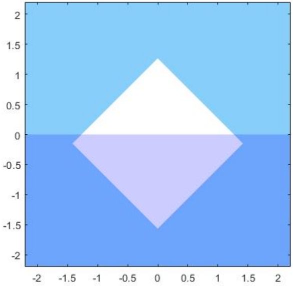




- Numerical solution and theory agree
- Stable equilibrium angle$= 0, 90, 180, 270$, or equivalent

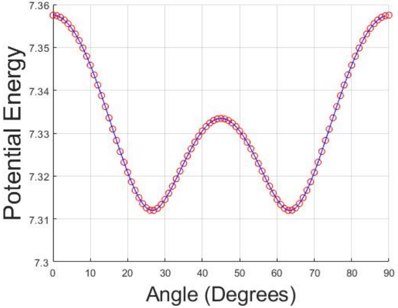
Potential Energy Landscape for Square when R=0.6

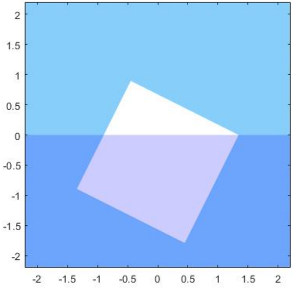




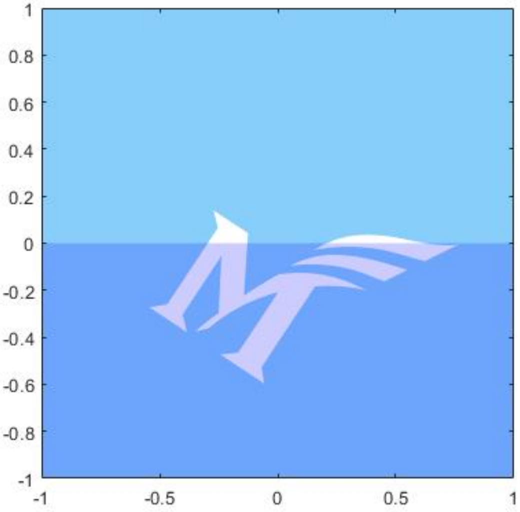
- Numerical solution and theory agree
- Stable equilibrium angle$= 45, 135$, or equivalent

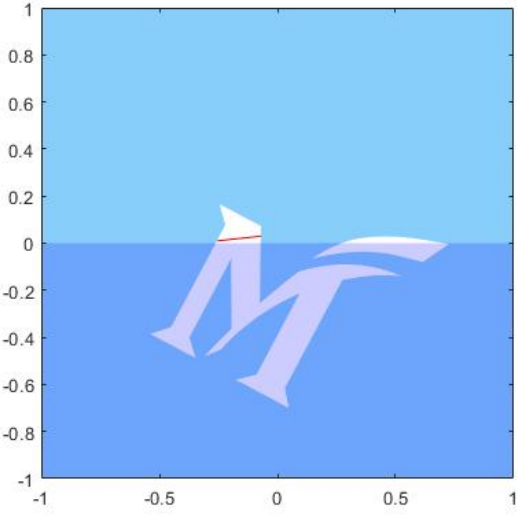
Potential Energy Landscape for Square when R=0.75





- Numerical solution and theory agree
- Stable equilibrium angle= $26.56, 63.44$,or equivalent


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.5
0
0.5
1


1
0.8
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-0.2
-0.4
-0.6
-0.8
-1
-1
-0.5
0
0.5
1

Next Steps

- Testing simple shapes: circle, ellipse, square, rectangle...
- Relationships between aspect ratios and the number of favorable orientations
- Metacenter
- Concavity, vertices, and invariant geometries
- Incorporate shape evolution

Acknowledgements

We would like to thank our project mentor, Dr. Anderson, and MEGL for supporting our research.

References

Gilbert, E. N. “How Things Float.” The American Mathematical Monthly, vol. 98, no. 3, 1991, p. 201., https://doi.org/10.2307/2325023.

Pollack, Henry. “Tip of the Iceberg.” Physics Today, vol. 72, no. 12, 2019, pp. 70–71., https://doi.org/10.1063/pt.3.4373.