Visualizing Geometric Structures

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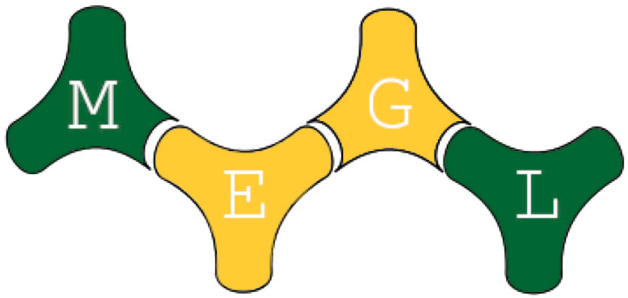


Main Idea of Project

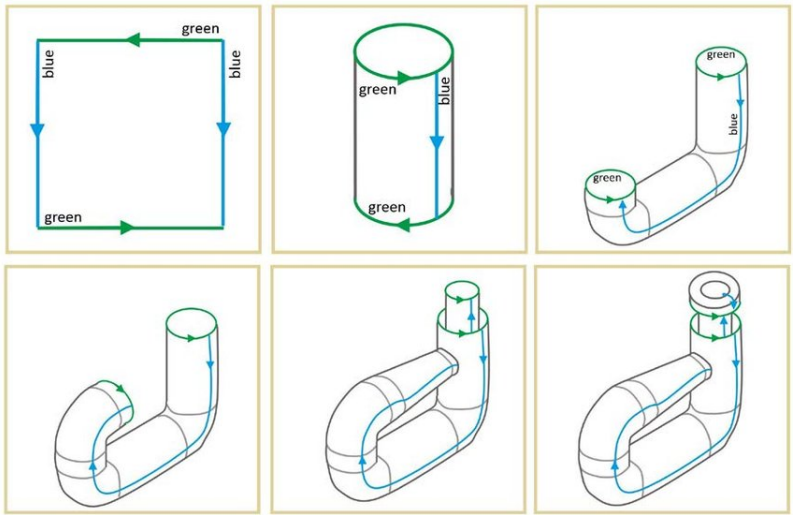
- The purpose of this project as a whole was to make it easier for people to understand some abstract mathematical structures.
- The Mathematics Visualization project has two sub-projects attached to it. One is called Snakes in the Plane and the other Hyperbolic Soccer.

Recap on Snakes on the Plane

- The snakes in the plane games was where we turned geometric surfaces into planes where the borders represented "glued" edges that would translate the snake and flip it over.
- Surfaces like the Mobius Strip or the Klein Bottle behave differently compared to the "normal" Euclidean plane, yet we find it difficult to imagine life on these sorts of surfaces because it is a global concept while we see our world from a local perspective.
- Snakes in the Plane allows us to see how life on these unique geometric structures differs from life on a Euclidean plane.

Translation Surfaces

- A translation surface is a surface which can be obtained by joining together the sides of a polygon.
- As an example, we can create a torus by attaching the opposite sides of a rectangle together.
- These surfaces are the settings for the scenes in Snakes in the Plane.



Progress Made on Snakes in the Plane

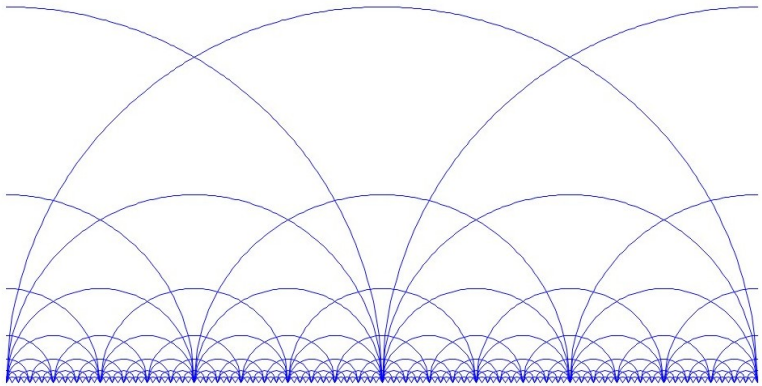
- First we added a scoreboard to the different Unity Scenes, so that the user could understand how they were progressing in the game.
- We added more levels to better show what it means to glue the edges of the rectangular plane. For example the cylinder level.
- In addition we made a dodecagon level

Gameplay

- Cylinder Level Gameplay - Simulates the surface of a cylinder. The snake can move on the sides freely but cannot move up and down.
- Dodecagon Gameplay - Simulates the surface of a structure created by attaching the opposite sides of a twelve sided polygon.

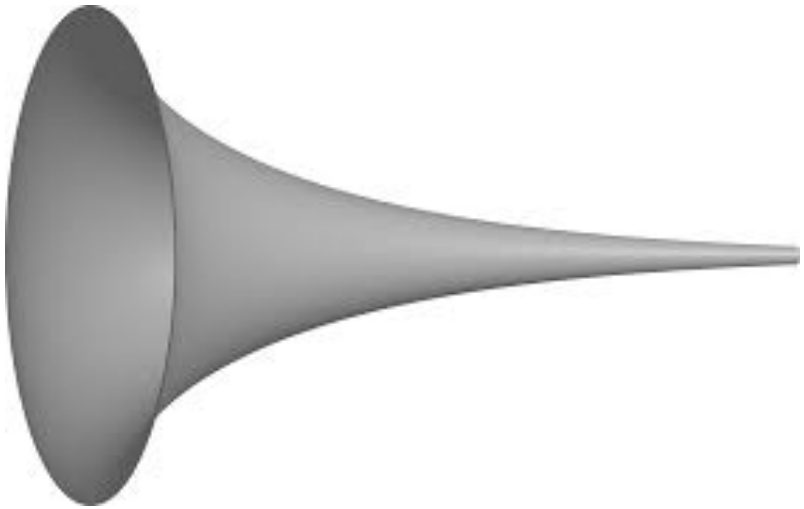
Recap on Hyperbolic Soccer

- The idea of Hyperbolic Soccer is to visualize linear movement on the hyperbolic plane.
- You try to fire a ball through a goal at the opposite end of the screen by sending it in some initial direction. Being on a hyperbolic plane, straight lines don't behave as you'd expect.
- The movement code uses the Poincaré half-plane model, a stereographic projection of a half sphere, which models linear hyperbolic movement as euclidean half-circles.



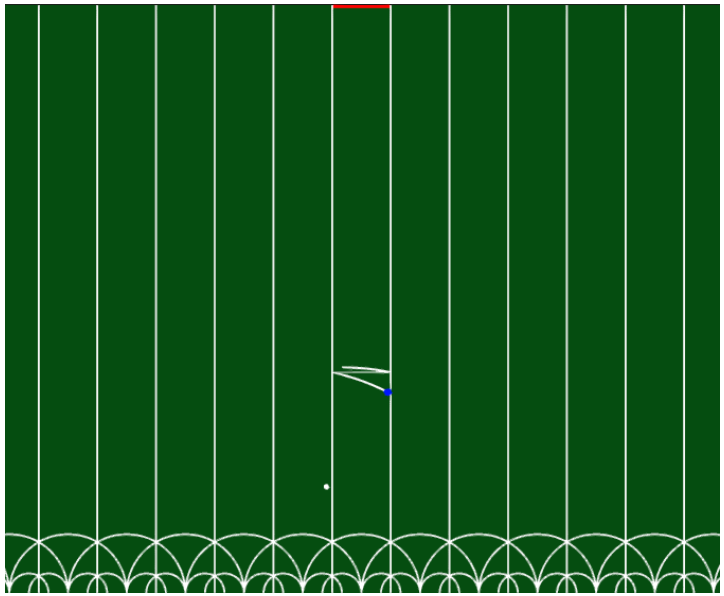
Hyperbolic Geometry

- Hyperbolic space is a space with constant negative curvature.
- Similarly to how world maps model spherical space on a euclidean plane, the Poincaré half-plane model models hyperbolic space on a euclidean plane.
- This is much more difficult to visualize. While its easy to think of a sphere to visualize spherical space, there is no way to place ALL of the hyperbolic plane in three-dimensional space.



Progress Made

- Bug fixes involving the camera and pointer (which shows the direction you are aiming).
- Improved background, which made the idea behind the game more obvious. Essentially drew on some possible paths which resembled the structure of the half-plane model.
- Added a trail to the ball's movement.
- Major aesthetic improvements.



Conclusion

- At the end of this semester, we had two main goals for this project. The first was to improve what was provided to us and the second was to learn some interesting mathematics along the way. We believe that we have done both.