The Geometry of Self-Driving Cars

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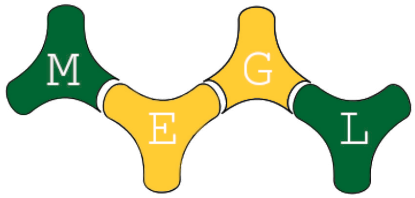
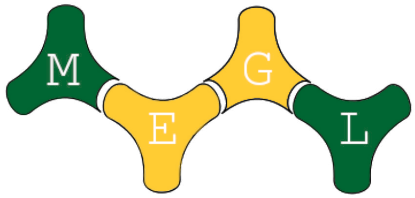


Introduction

- As Self-Driving cars are becoming more popular, there is a need to minimize the length or time in order to minimize cost. For this, we must first understand the geometry of Self-Driving cars

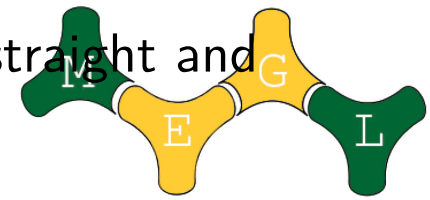
- Since Self-Driving cars have a turning radius, we cannot use Euclidean distance to measure path-length. This problem resembles what Lexter Eli Dubins described in his 1957's research paper, *On Curves of Minimal Length with a Constraint on Average Curvature, and with Prescribed Initial and Terminal Positions and Tangents*, a paper in which he explained the possible optimal paths between two points given a constraint on curvature.



Dubins Model Overview

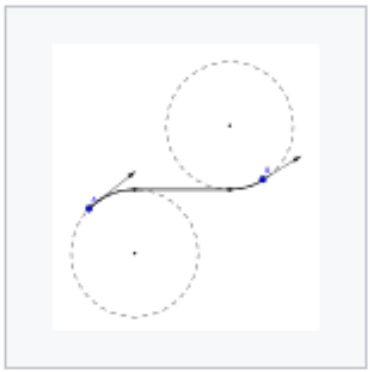
- The Dubins model is commonly used in the fields of robotics and control theory as a way to plan paths for wheeled robots, airplanes and underwater vehicles, and usual driving - especially at high speed.
- An object can directly move forward only. To let objects move backwards, we would have to apply Reed-Shepp's algorithm which we will not discuss here.
- It can get anywhere if there are no obstacles; otherwise, it will get stuck.
- It cannot wiggle to change the angle.

The most effective way to get to the destination is to combine straight and full-turn motions.

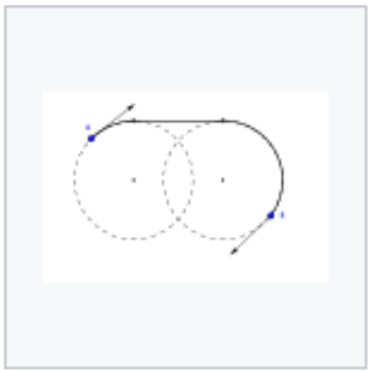


Mathematical Description of Dubins' Model

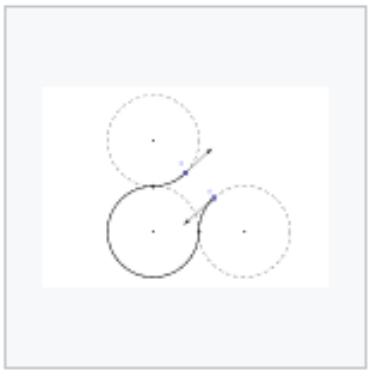
If an object is moving from $P_1(x_1, y_1, \theta_1)$ to $P_2(x_2, y_2, \theta_2)$ given a turning radius constraint on a continuous and differentiable path, what should be the shortest path?



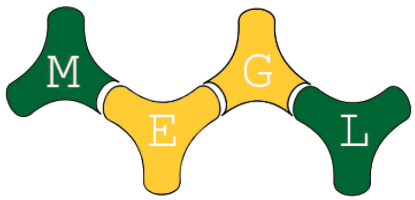
An RSL Dubins path



An RSR Dubins path



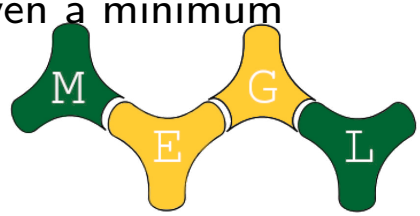
An LRL Dubins path



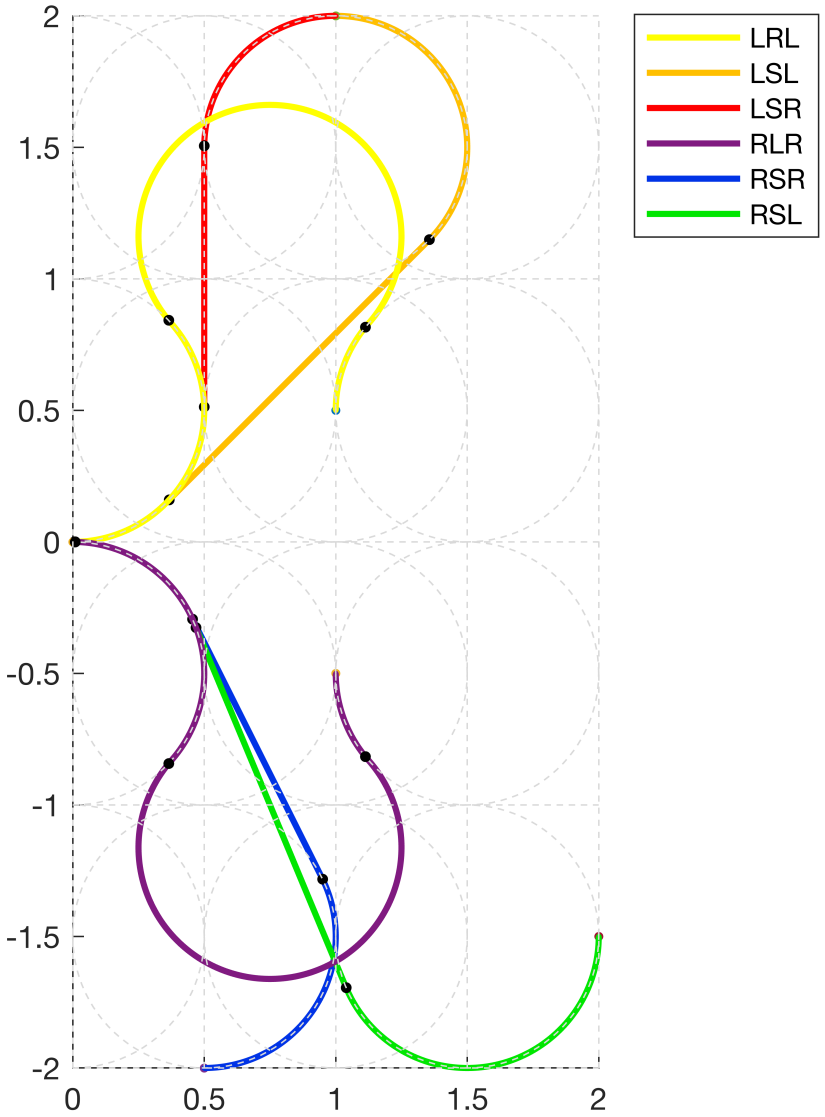
Achievements

In our project we aimed to study the geometry of Dubins space. Using Matlab's and The Open Motion Planning Library (OMPL)'s DubinsStateSpace packages we were able to:

- Analyze Dubin's paths and validate Dubin's 1957 research paper based on these observations.
- Calculate path lengths and observe length changes as the initial and final directions change when having fixed initial and final position points:
  $f : \{(\theta_1, \theta_2) : \theta_i \in \{0, \frac{\pi}{n}, \frac{2\pi}{n}, \frac{2n\pi}{n}\}\} \rightarrow$ Shortest Path
- Categorize Dubin's paths, where:
  $f : \{\mathbb{R}x\mathbb{R}x[0, 2\pi]\} \rightarrow S = \{LRL, LSL, LSR, RLR, RSR, RSL\}$ where
  $\{L, R, S, LS, LR, RL, RS\} \subset S$
- Create a dynamic tool to observe the behavior of these short paths given a minimum turning radius, destination coordinates, and direction points.

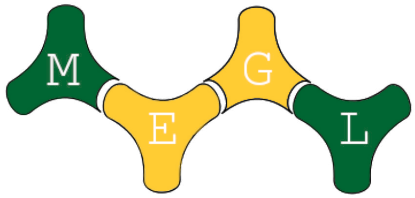


Dubins Paths Consist of CSC & CCC Trajectories



Video of Dubins' Trajectories

Click Here To Watch Dubins' Trajectories Video

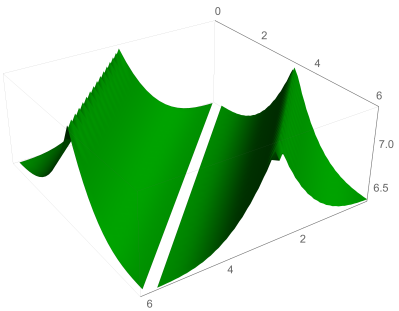


Dubins Length - Graph of the $(0,0)$ to $(0,0)$ Slice

For this example we used fixed initial and final positions (x_1, y_1), (x_2, y_2) at $(0,0)$, $(0,0)$ respectively. The coordinates of this graph represents $(\theta_1, \theta_2, \textit{length})$, where θ_1 is the initial direction, θ_2 is the ending direction, length is the outcome of the function f defined by:

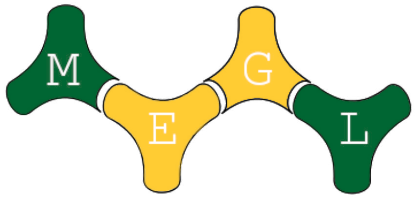
$$f(x_1, y_1, \theta_1, x_2, y_2, \theta_2) = f_{x,y}(\theta_1, \theta_2) = f : \{(\theta_1, \theta_2) : \theta_i \in \{0, \tfrac{\pi}{n}, \tfrac{2\pi}{n}, \tfrac{2n\pi}{n}\}\} \rightarrow$$
$$\text{length} = \text{Dubins Shortest Path}$$

This graph has a natural discontinuity when $\theta_1 = \theta_2$. This is because the position and direction would not change at these points; however, when these points do not lie on the $\frac{\pi}{4}$ line, all points are continuous. Notice both, "upper" and "lower", regions in reference to the 45 degree line - these regions are continuous but may not necessarily be differentiable.

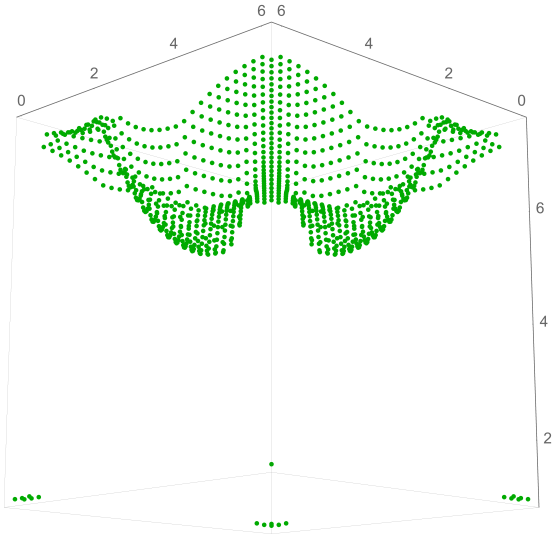


Video of Dubins'$(0, 0, \theta_1)$ to $(0, 0, \theta_2)$ Slice

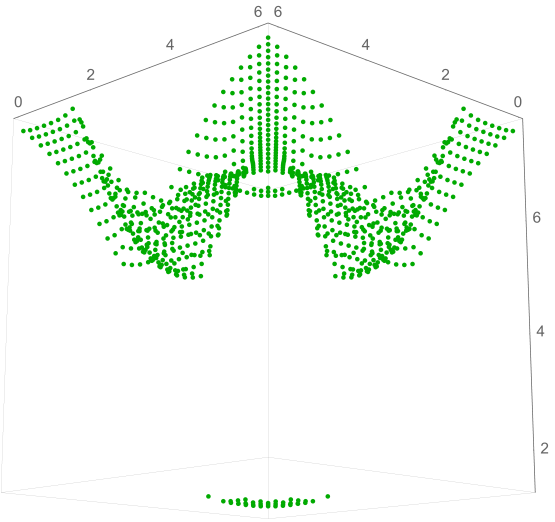
Click Here To Watch Dubins' $(0, 0, \theta_1)$ to $(0, 0, \theta_2)$ Slice



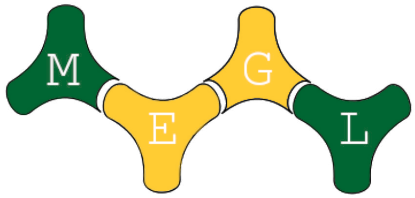
Other Interesting Slices



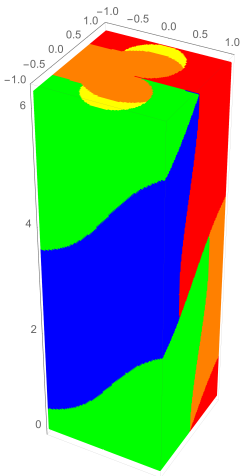
$(0, 0)$ to $(1, 0)$



$(0, 0)$ to $(1, 1)$



Dubins Paths Categorized by Minimum Turning Radius



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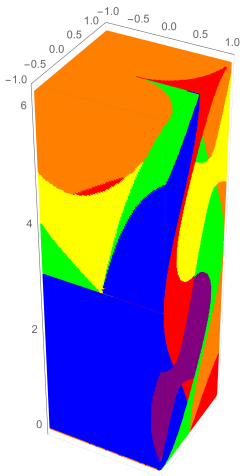

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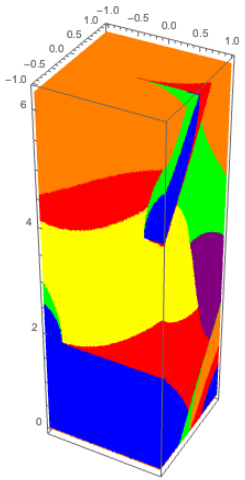
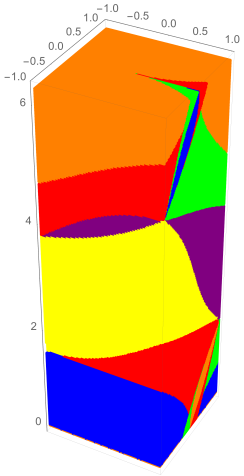
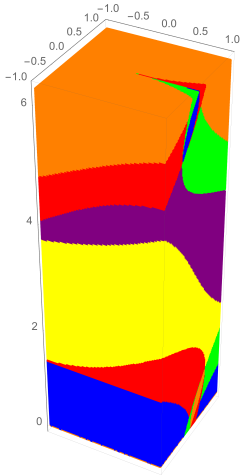


0.75



1.0



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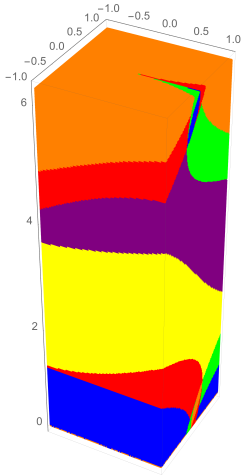

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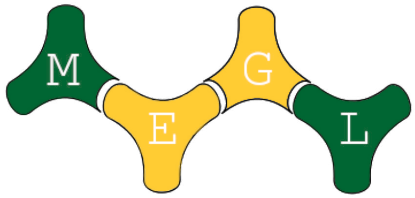


Code Behind

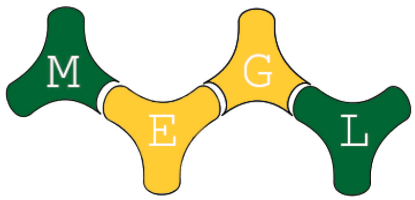
Below is a brief description of the algorithms derived from Dubins observations. We used Matlab's DubinsStateSpace package to retrieve path data for each (x_1, y_1, θ_1) to (x_2, y_2, θ_2), looped through a given space, categorized path types, create images and videos for analysis purposes - See Path Algorithm. We retrieved the Arc Length Algorithm from other documentation sources, length is a built-in function under DubinsStateSpace.

Path Algorithm

Let d = ""
For i = 2 to Rows From Interpolated Data
 $\theta_i = atan2(y_i - y_{i-1}, x_i - x_{i-1})$
 If $\theta_{i-1} - \theta_i = 0$ Then
  d = "straight"
 Else If $theta_i > theta_{i-1}$ Then
  d = "right"
 Else
  d = "left"
 End If
 If $(|\theta_i - \theta_{i-1}| > \pi)$
  If d = "right" Then
   d = "left"
  Else If d = "left" Then
   d = "right"
  End If
 End If
d = d + d
End For
Return d

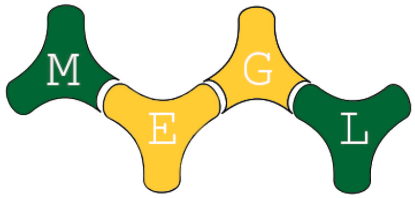
Arc Length Algorithm

$\theta = atan2(y_i - y_{i-1}, x_i - x_{i-1})$
If $\theta =< 0$ And d ="left" Then
 $\theta = \theta + 2\pi$
else If $\theta > 0$ And d ="right" then
 $\theta = \theta - 2\pi$
End If
Return $|(|\theta * r)$



Conclusion & Future Goals

We concluded that Dubins space is discontinuous. This was expected because Dubins space is asymmetrical. We also concluded that Dubins paper is correct based on experimental observations. As future work, we would like to analyze the Dubins slices (e.g., the $(0, 0, \theta_1)$ to $(0, 0, \theta_2)$ slice we previously shown), we would also like to understand the 3D blocks better, and improve any tools built during this semester.



Thank you!