Visualizing Bruhat-Tits Buildings

Bram Bekker

MEGL, George Mason University
Radboud University, Nijmegen, The Netherlands
Under supervision of:
Dr. Maarten Solleveld
Dr. Anton Lukyanenko

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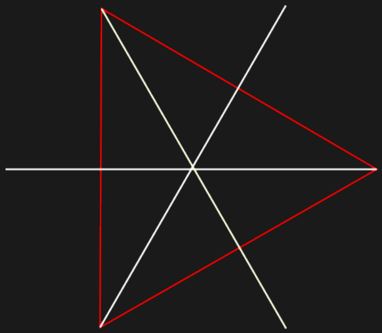
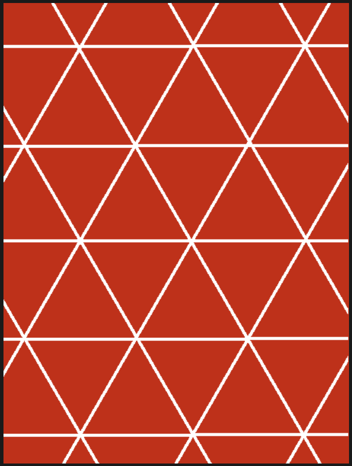
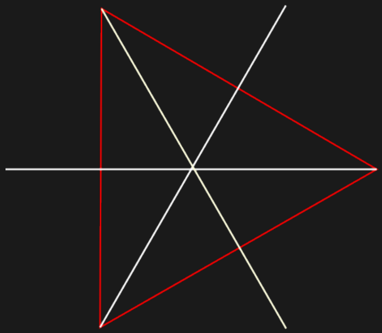
Buildings

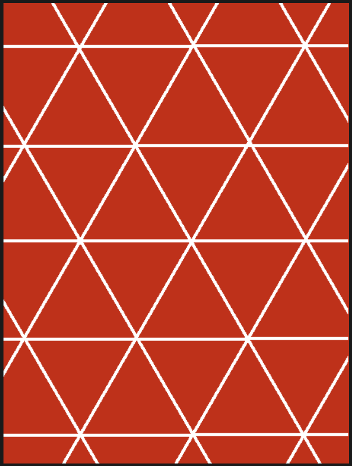




Buildings



S_3



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On rationals: $\nu(a/b) = \nu(a) - \nu(b)$

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B is in bijection with

$$T(\nu = 0) \prod_{i<j} U_{ij}(\nu \geq 0) \prod_{i>j} U_{ij}(\nu \geq 1)$$

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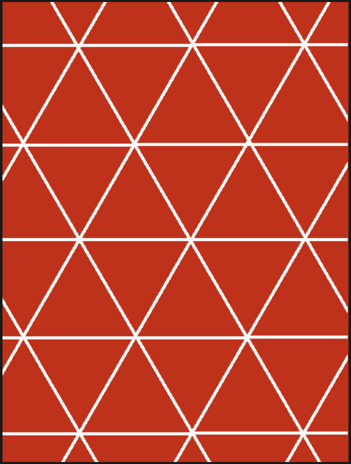

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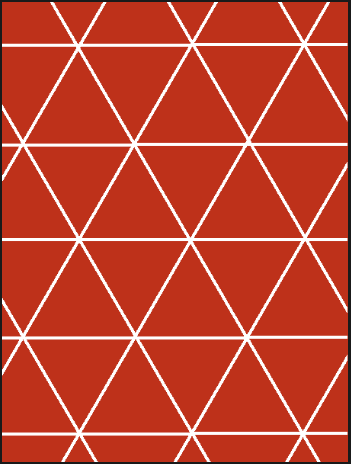
We look for a bigger building where chambers are identified with *cosets* of B



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$$B/\left(B \cap wBw^{-1}\right) \cong \prod_{i<j} \mathbb{Z}/2^{k_{ij}}\mathbb{Z} \prod_{i>j} 2\mathbb{Z}/2^{k_{ij}}\mathbb{Z}$$