A Hero's Journey: A Tale of Polyhedral Cones

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Why do anything?

- Our objects of interest are algebraic varieties, and in order to get some idea of what the coordinate ring of our variety looks like, we tropicalize the variety and get a corresponding polyhedral cone. This is easier to study, and finding what are called the Hilbert and Markov bases of the cone gets us an upper bound on the presentation of the coordinate ring of our variety.
- In order to form a polyhedral cone, we take a graph, assign to each vertex a Berenstein-Zelevinsky triangle, and let the edges denote a connection between vertex triangles. This forms a polyhedral cone using the vertices of the triangles as coordinates.

Hilbert and Markov Bases

- The Hilbert basis of a polyhedral cone is a set of vectors that span the integer lattice of the cone. In other words, if you want to go to some integer point in the cone, the Hilbert basis is a list of possible vectors you can use to get there.
- The Markov basis describes the relationship between the elements of the Hilbert basis. If the HB elements inherit the addition structure of integer vectors, a linear combination of some elements might equal a different combination of HB elements. This data is taken as an element of the Markov basis of the cone.

Trees

- This semester, we focused on adapting our computer code to interpret trivalent trees in addition to the trivalent pseudo graphs we had analyzed over the summer. Also, we added a condition to the trees, whereby we set a boundary term on the leaf vertices of a tree, or left those vertices "free."

Table

- The table, on the MEGL website in the polytopes summer 2016 research tab, is organized into 3 distinct columns. These correspond to the pseudo-graphs and trees we have so far found Hilbert and Markov bases for. The trees are divided into 2 categories; bound and free, which are distinguished by color.
- Under each picture of a graph, we put links to the HB and MB for each SL_n type of triangle.

DA FUTURE

- We are planning to note the time that each computation of HB or MB takes, and we've noted so far that for some trees, the computations take much longer than for others.